# Invisibles 2013
# Cosmology - Lecture 1


## Ed Copeland -- Nottingham University


1. The general picture, evolution of the universe: assumptions and evidence supporting them.

2. Dark Energy - Dark Matter - Modified Gravity

3. Origin of  Inflation and the primordial density fluctuations.

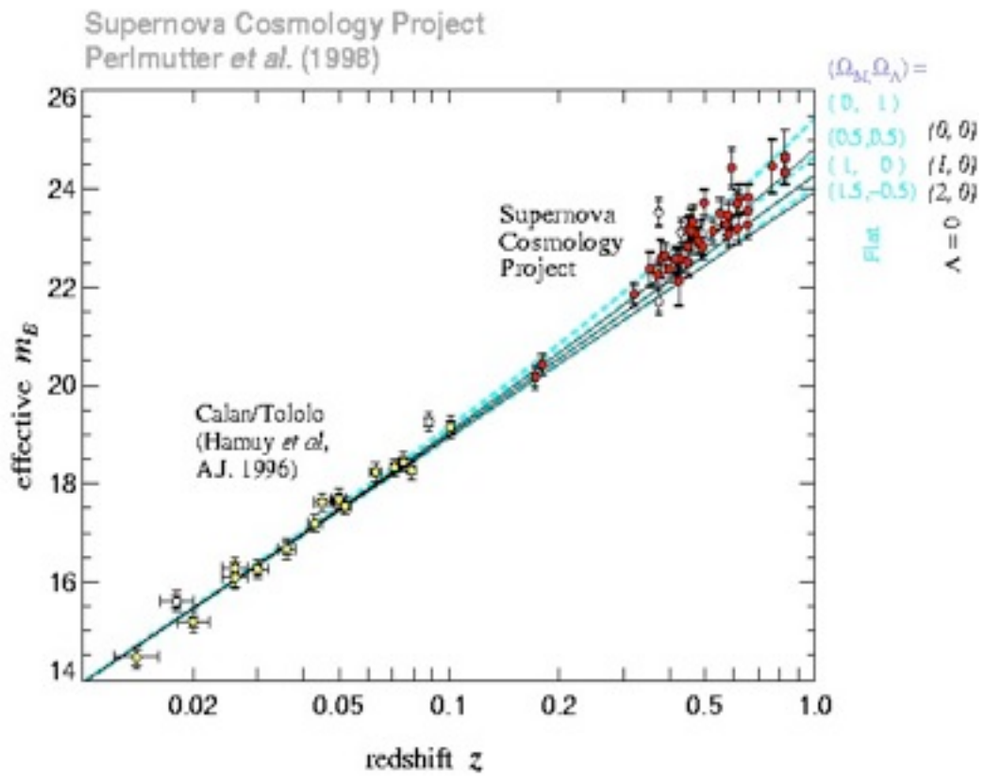
Related Lecture notes : http://www.nottingham.ac.uk/~ppzejc/cosmology/ModCosm_notes.pdf

## July 10 - 11, 2013

## Durham University

# 1. The Big Bang – (1sec → today)

The cosmological principle -- isotropy and homogeneity on large scales




**Test 1**

- **The expansion of the Universe**
  **$v=H_0 d$**

**$H_0=74.2\pm3.6$ km s$^{-1}$ Mpc$^{-1}$**

**(Riess et al, 2009)**

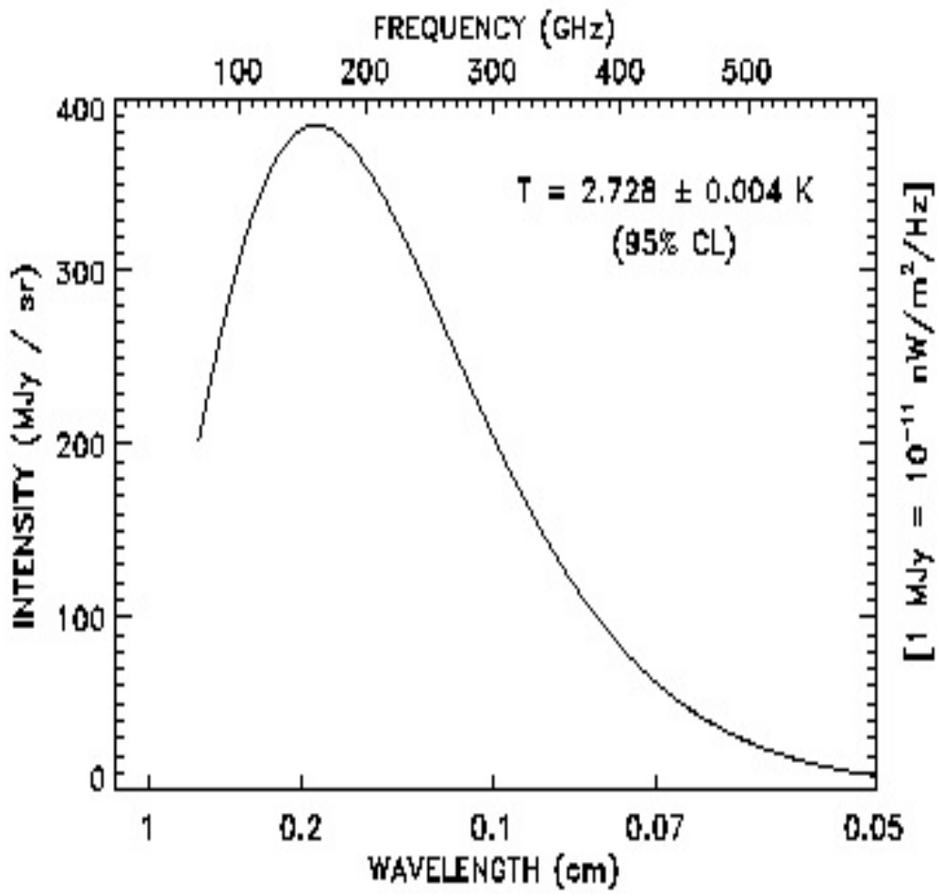
Distant galaxies receding with vel proportional to distance away.

Relative distance at different times measured by scale factor a(t) with

$$H = \frac{\dot{a}}{a}$$

- Nobel prize for Saul Perlmuter, Brian Schmidt and Adam Riess in 2011

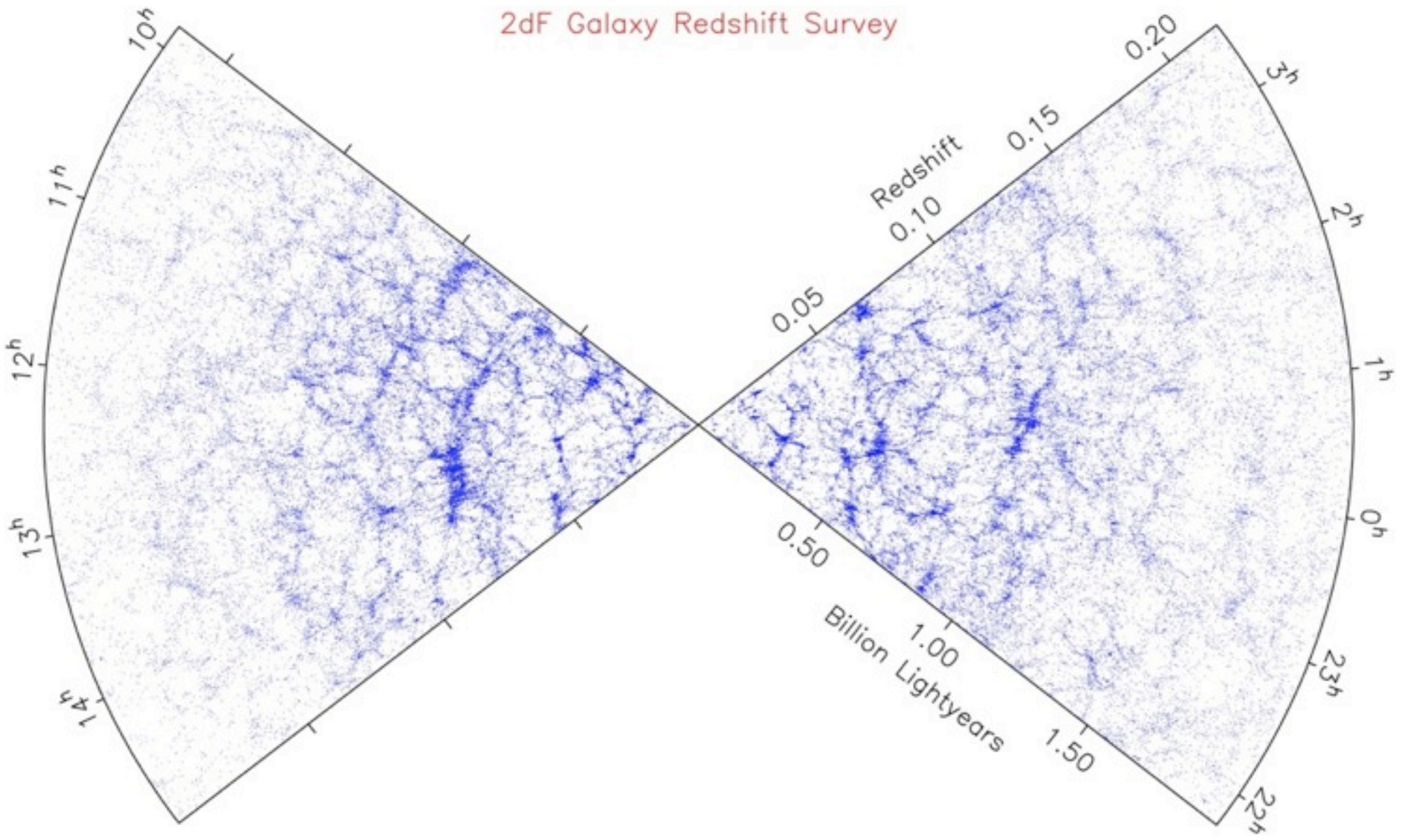
# The Big Bang – (1sec → today)




**Test 2**

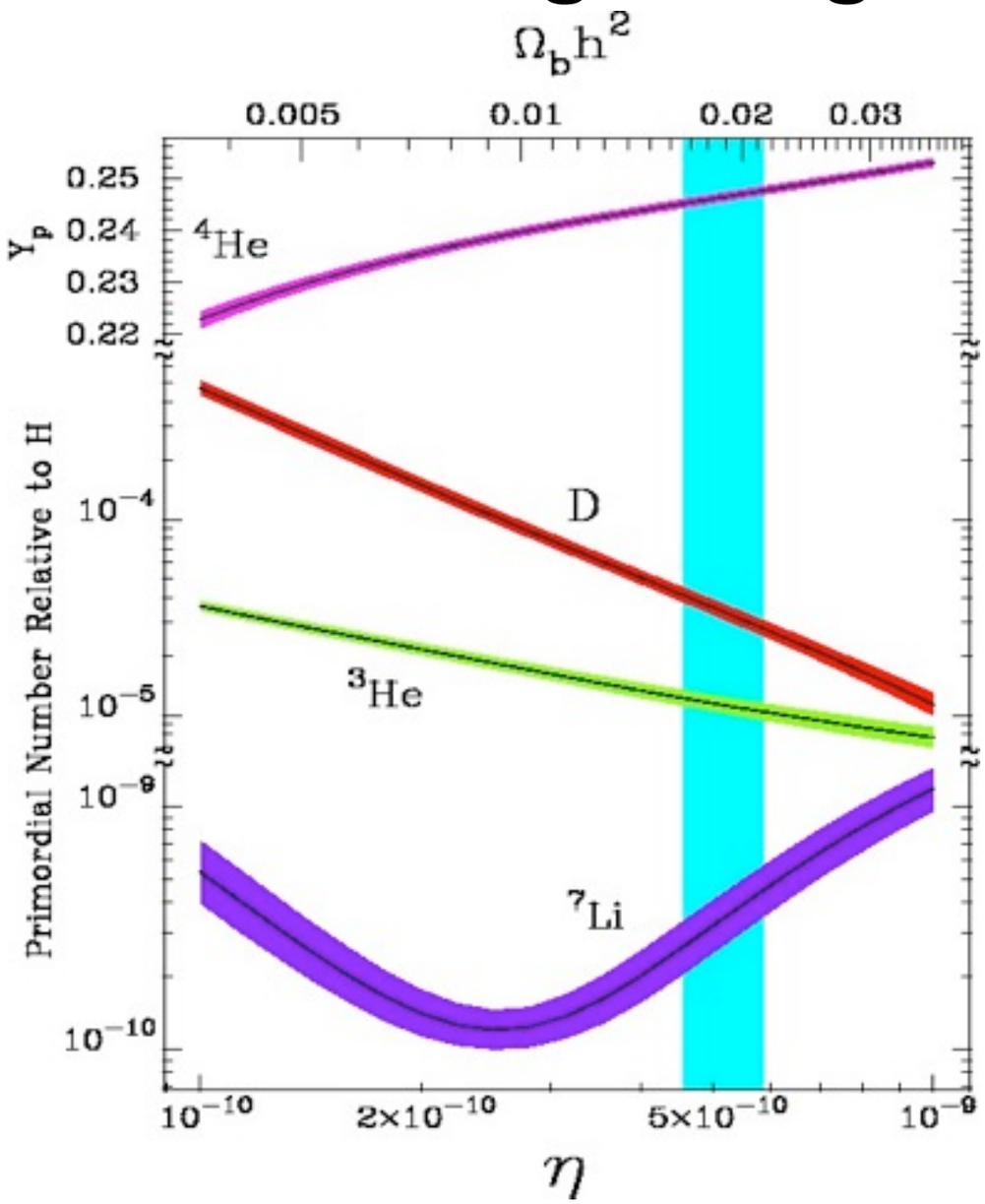
- **The existence and spectrum of the CMBR**
- **$T_0$=2.728 ± 0.004 K**
- Evidence of isotropy -- detected by COBE to such incredible precision in 1992
- Nobel prize for John Mather 2006

# 2dF Galaxy Redshift Survey




Homogeneous on large scales?

# The Big Bang – (1sec → today)




**Test 3**

- **The abundance of light elements in the Universe.**
- **Most of the visible matter just hydrogen and helium.**

WMAP7 - detected effect of primordial He on temperature power spectrum, giving new test of primordial nucleosynthesis.

$$Y_P = 0.326 \pm 0.075$$

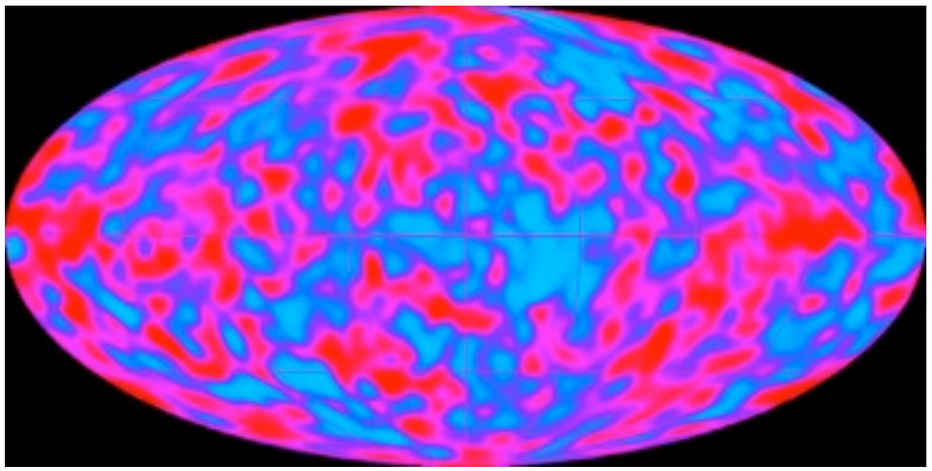
**(Komatsu et al, 2010)**

$$\Omega_b h^2 = 0.0225 \pm 0.0005 \ (68\% \ CL)$$

# The Big Bang – (1sec → today)

## Test 4

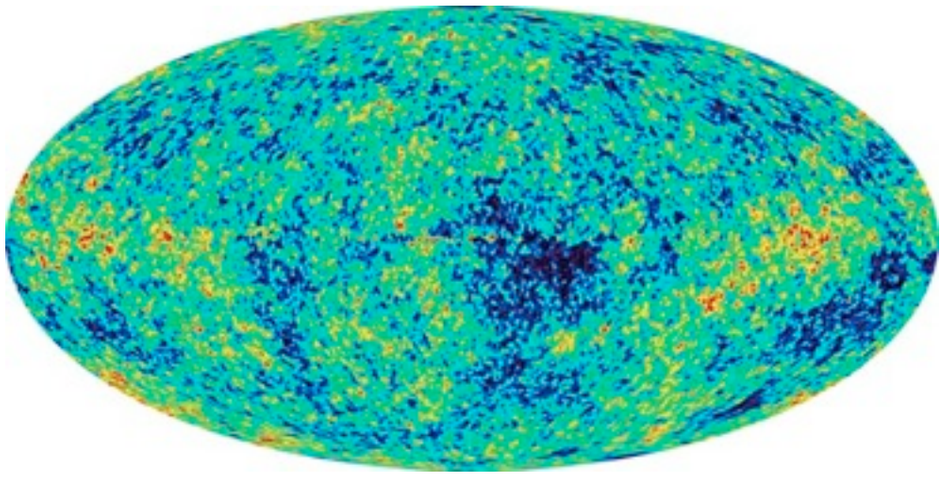
- **Given the irregularities seen in the CMBR, the development of structure can be explained through gravitational collapse.**



**COBE - 1992, 2006**

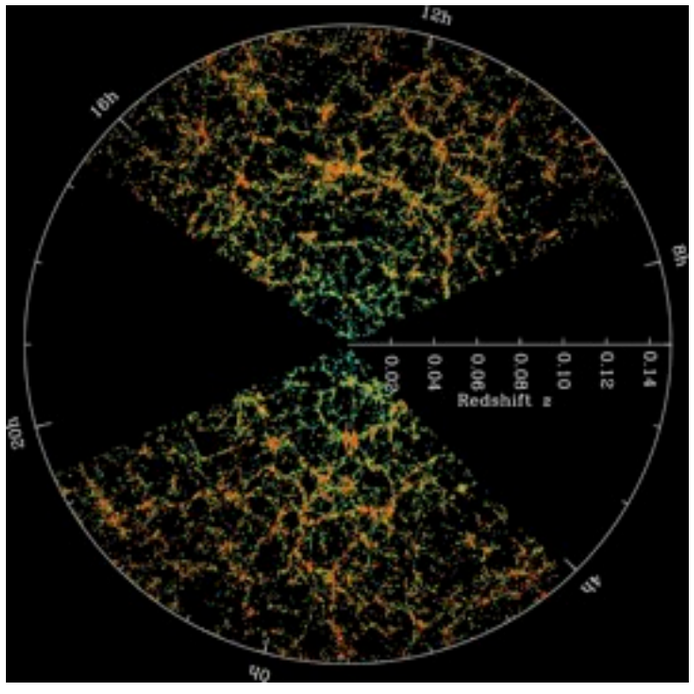
**Nobel prize for**

**George Smoot**



**WMAP-2010**

**SDSS**



# The key equations

Einstein GR: $$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$

Geometry — Matter — Cosm const - could be matter or geometry

Relates curvature of spacetime to the matter distribution and its dynamics.

Require metric tensor $g_{\mu\nu}$ from which all curvatures derived indep of matter:

Invariant separation of two spacetime points (μ,ν=0,1,2,3): $$ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$$

Einstein tensor $G_{\mu\nu}$ -- function of $g_{\mu\nu}$ and its derivatives.
Energy momentum tensor $T_{\mu\nu}$ -- function of matter fields present.
For most cosmological substances can use perfect fluid representation for which we write $$T_{\mu\nu} = (\rho + p) U_\mu U_\nu + p g_{\mu\nu}$$

$U^\mu$ : fluid four vel = (1,0,0,0) - because comoving in the cosmological rest frame.
(ρ,p) : energy density and pressure of fluid in its rest frame

$$T_{\mu\nu} = \mathrm{diag}(\rho, p, p, p)$$

# Reminder of curvatures

Christoffel symbols: $$\Gamma^{\mu}_{\nu\sigma} = \frac{1}{2} g^{\mu\lambda} (g_{\sigma\lambda,\nu} + g_{\nu\lambda,\sigma} - g_{\sigma\nu,\lambda})$$

Riemann's curvature tensor: $$R^{\mu}_{\nu\sigma\gamma} = \Gamma^{\mu}_{\nu\gamma,\sigma} - \Gamma^{\mu}_{\nu\sigma,\gamma} + \Gamma^{\mu}_{\alpha\sigma}\Gamma^{\alpha}_{\gamma\nu} - \Gamma^{\mu}_{\alpha\gamma}\Gamma^{\alpha}_{\sigma\nu}$$

Ricci tensor: $$R_{\mu\nu} = R^{\sigma}_{\mu\nu\sigma}$$

Ricci scalar: $$R = R^{\mu}_{\mu}$$

Einstein tensor: $$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$$

Not needed here -- maybe in the tutorials

# Cosmology - isotropic and homogeneous FRW metric

Copernican Principle: We are in no special place. Since universe appears isotropic around us, this implies the universe is isotropic about every point. Such a universe is also homogeneous.

Line element $$ds^2 = -dt^2 + a^2(t)dx^2$$

$$dx^2 = \frac{1}{1-kr^2}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

t -- proper time measured by comoving (i.e. const spatial coord) observer.
a(t) -- scale factor: k- curvature of spatial sections: k=0 (flat universe), k=-1 (hyperbolic universe), k=+1 (spherical universe)

Aside for those familiar with this stuff -- not chosen a normalisation such that $a_0$=1. We are not free to do that and simultaneously choose |k|=1. Can do so in the k=0 flat case.

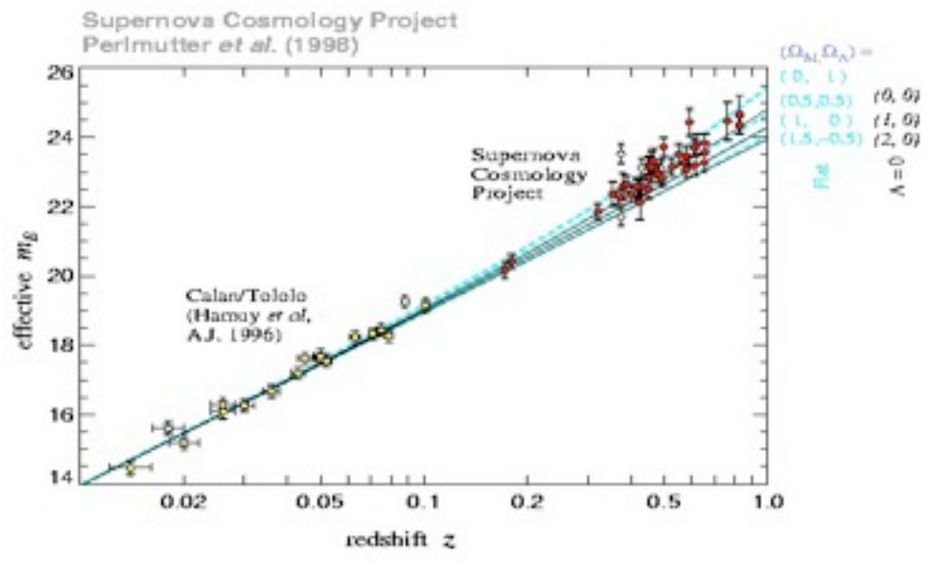
Intro Conformal time : τ(t) $$\tau(t) \equiv \int^{t} \frac{dt'}{a(t')}$$

Implies useful simplification : $$ds^2 = a^2(\tau)(-d\tau^2 + dx^2)$$

Hubble parameter :
(often called Hubble constant) $$H(t) \equiv \frac{\dot{a}}{a}$$

Hubble parameter relates velocity of recession of distant galaxies from us to their separation from us

$$v = H(t)r$$



$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu}$$ applied to cosmology

Friedmann:

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2} + \frac{\Lambda}{3}$$

a(t) depends on matter, $\rho(t)=\Sigma_i \rho_i$ -- sum of all matter contributions, rad, dust, scalar fields ...

Energy density $\rho(t)$: Pressure $p(t)$

Related through : $p = w\rho$

Eqn of state parameters: w=1/3 – Rad dom: w=0 – Mat dom: w=-1– Vac dom

**Eqns (Λ=0):**

**Friedmann + Fluid energy conservation**

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\nabla^\mu T_{\mu\nu} = 0$$

# Combine Friedmann and fluid equation to obtain Acceleration equation:

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G\,(\rho + 3p) \quad --- \; Accn$$

$$If \;\; \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

Inflation condition -- more later

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$\dot{\rho} + 3(\rho + p)\frac{\dot{a}}{a} = 0$$

$$\rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3(1+w)} \quad ; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3(1+w)}}$$

$$\text{RD} \;\; : \;\; w = \frac{1}{3} : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-4} \;\; ; \; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{1}{2}}$$

$$\text{MD} \;\; : \;\; w = 0 : \rho(t) = \rho_0 \left(\frac{a}{a_0}\right)^{-3} \;\; ; \; a(t) = a_0 \left(\frac{t}{t_0}\right)^{\frac{2}{3}}$$

$$\text{VD} \;\; : \;\; w = -1 : \rho(t) = \rho_0 \;\; ; \; a(t) \propto e^{Ht}$$

Solutions with curvature in problem set.

# A neat equation

$$\rho_c(t) \equiv \frac{3H^2}{8\pi G} \quad ; \quad \Omega(t) \equiv \frac{\rho}{\rho_c}$$

$$\Omega > 1 \leftrightarrow k = +1$$
$$\Omega = 1 \leftrightarrow k = 0$$
$$\Omega < 1 \leftrightarrow k = -1$$

Friedmann eqn

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

$\Omega_m$ - baryons, dark matter, neutrinos, electrons, radiation ...

$\Omega_\Lambda$ - dark energy ; $\Omega_k$ - spatial curvature

$$\rho_c(t_0) \equiv 1.88 h^2 * 10^{-29} \, g cm^{-3}$$ Critical density

Current bounds on H(z) -- Komatsu et al 2010 - (WMAP7+BAO+SN)

$$H^2(z) = H_0^2 \left( \Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_{de} \exp\left( 3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right) \right)$$

(Expansion rate) -- $H_0$=70.4 ± 1.3 km/s/Mpc

(radiation) -- $\Omega_r = (8.5 \pm 0.3) \times 10^{-5}$

(baryons) -- $\Omega_b = 0.0456 \pm 0.0016$

(dark matter) -- $\Omega_m = 0.227 \pm 0.014$

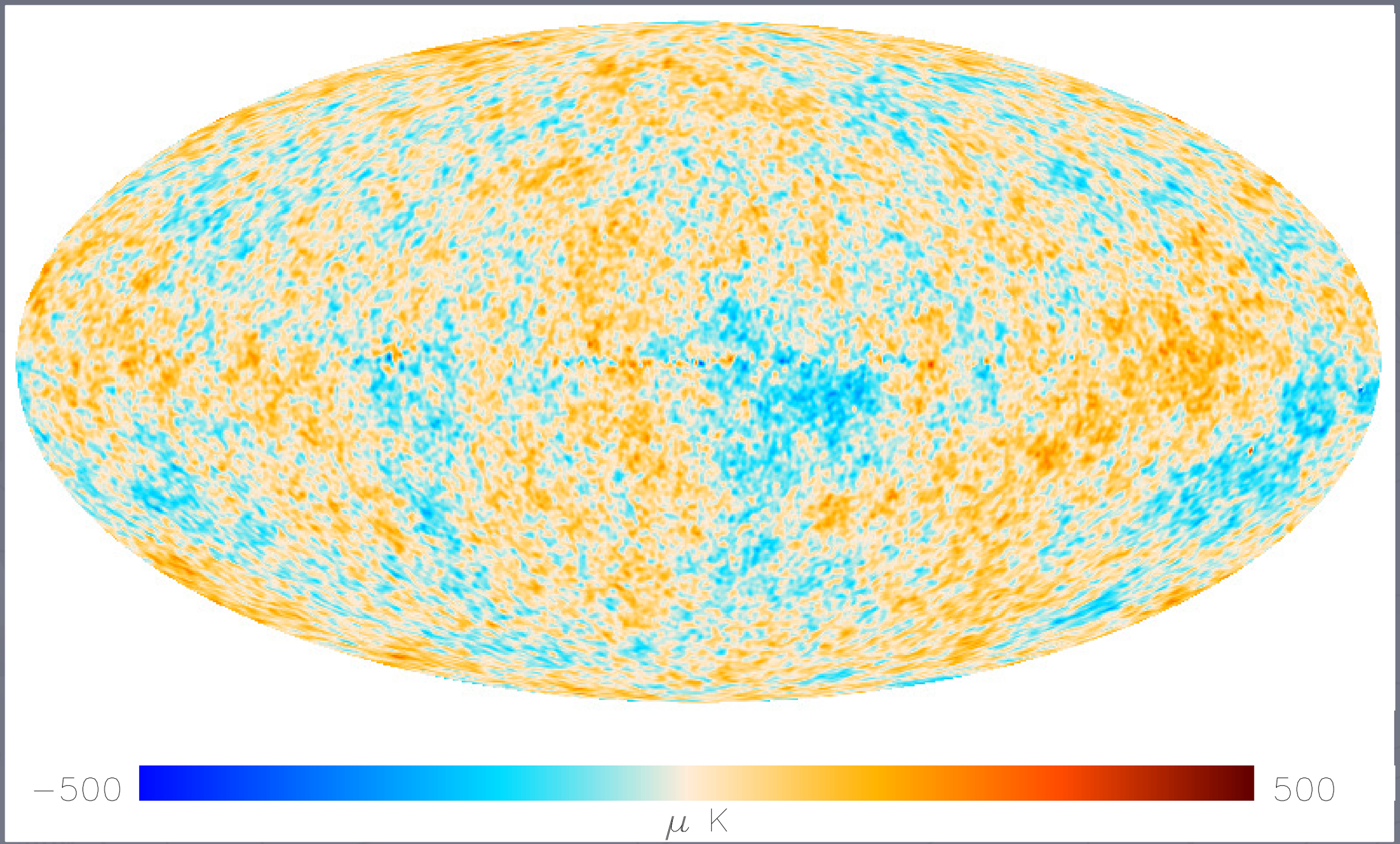
(curvature) -- $\Omega_k < 0.008$ (95%CL)

(dark energy) -- $\Omega_{de} = 0.728 \pm 0.015$ -- Implying univ accelerating today

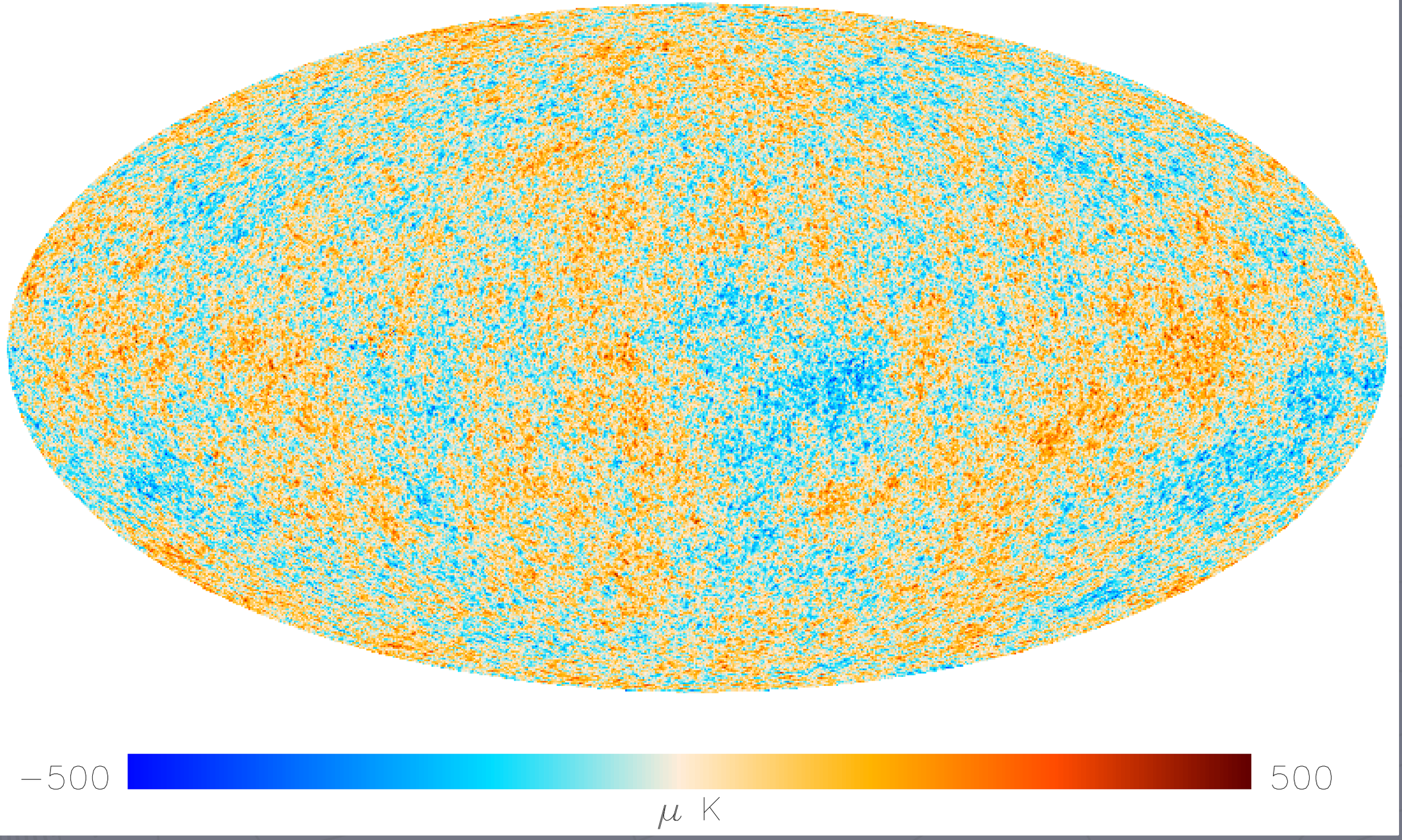
(de eqn of state) -- 1+w = 0.001 ± 0.057 -- looks like a cosm const.

If allow variation of form : $w(z) = w_0 + w' z/(1+z)$ then
$w_0$=-0.93 ±0.12 and w'=-0.38 ± 0.65 (68% CL)

# WMAP 9 year



# Planck - 1 year - wow !




- Improvement over WMAP: ang resolution (x2.5), sensitvity (x10), freq coverage [9 bands (30-857 GHz) v 5 bands (23-90 GHz)]

# How old are we?

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi}{3} G\rho - \frac{k}{a^2}$$

$$where\ \rho = \rho_m + \rho_r + \rho_\Lambda$$

$$t = \int \frac{da}{\dot{a}} = \int \frac{da}{aH}$$

$$t_0 = H_0^{-1} \int_0^1 \frac{x\ dx}{\left[\Omega_{m0} x + \Omega_{r0} + \Omega_{\Lambda 0} x^4 + (1-\Omega_0) x^2\right]^{1/2}}$$

$$where\ \Omega_0 = \Omega_{m0} + \Omega_{r0} + \Omega_{\Lambda 0}$$

$$Today: H_0^{-1} = 9.8 \times 10^9\ h^{-1}\ years;\ h = 0.7$$

$H_0^{-1}$ – –Hubble time

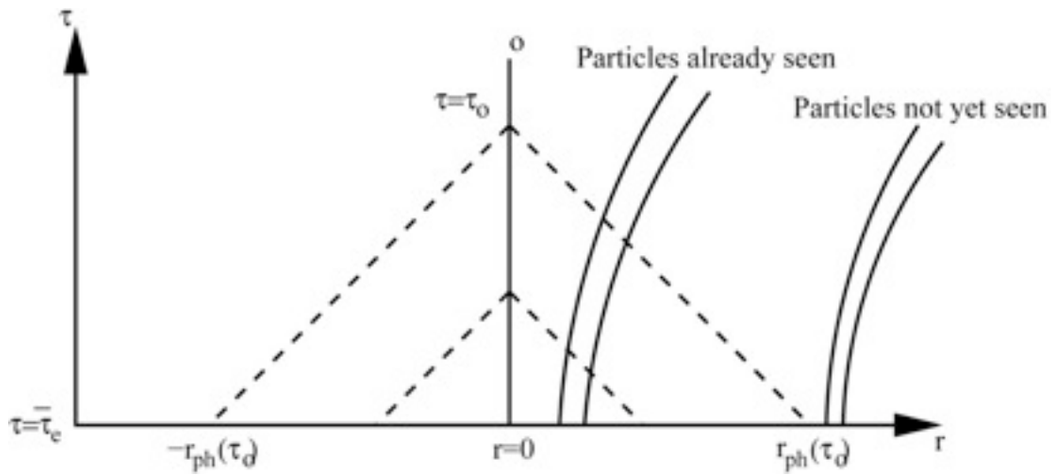
Useful estimate for age of universe

| $\Omega_{m0}$ | $\Omega_{r0}$ | $\Omega_{\Lambda 0}$ | $t_0$ |
|---|---|---|---|
| 1 | 0 | 0 | 9.4 Gyr |
| 0.3 | $10^{-5}$ | 0.7 | 13.4 Gyr |
| Open | | | |
| 0.2 | $10^{-5}$ | 0.2 | 12.4 Gyr |
| 0.2 | $10^{-5}$ | 0.6 | 13.96 Gyr |
| Closed | | | |
| 0.3 | $10^{-5}$ | 0.8 | 13.96 Gyr |
| 0.4 | $10^{-5}$ | 0.9 | 13.6 Gyr |

# Horizons -- crucial concept in cosmology

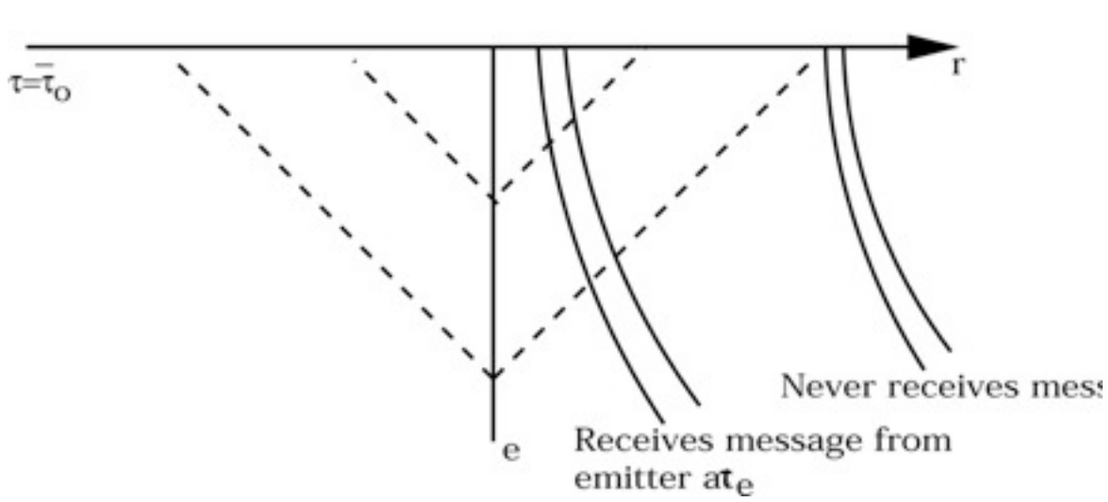
a) Particle horizon: is the proper distance at time t that light could have travelled since the big bang (i.e. at which a=0). It is given by

$$d_p(t) = a(t) \int_0^t \frac{dt'}{a(t')}$$




b) Event horizon: is the proper distance at time t that light will be able to travel in the future:

$$d_{\rm EH}(t) = a(t) \int_t^\infty \frac{dt'}{a(t')}$$




Trodden and Carroll 03

# History of the Universe

| | | | |
|---|---|---|---|
| $10^{18}$ GeV | $10^{-43}$ sec | $10^{32}$ K | QG/String epoch |
| | | | Inflation begins (?) |
| $10^{3}$ GeV | $10^{-10}$ sec | $10^{15}$ K | Electroweak tran |
| 1 GeV | $10^{-4}$ sec | $10^{12}$ K | Quark-Hadron tran |
| 1 MeV | 1 sec | $10^{10}$ K | Nucleosynthesis |
| 1 eV | $10^{4}$ years | $10^{4}$ K | Matter-rad equality |
| | $10^{5}$ years | $3.10^{3}$ K | Decoupling → microwave bgd. |
| $10^{-3}$ eV | $10^{10}$ years | 3K | Present epoch |

# The Big Bang – problems.

- Flatness problem – observed almost spatially flat cosmology requires fine tuning of initial conditions.
- Horizon problem -- isotropic distribution of CMB over whole sky appears to involve regions that were not in causal contact when CMB produced. How come it is so smooth?
- Monopole problem - where are all the massive defects which should be produced during GUT scale phase transitions.
- Relative abundance of matter – does not predict ratio baryons: radiation: dark matter.
- Origin of the Universe – simply assumes expanding initial conditions.
- Origin of structure in the Universe from initial conditions homogeneous and isotropic.
- The cosmological constant problem.

# Flatness problem

$$\Omega^{-1} = 1 - \frac{3k}{\kappa^2 \rho a^2} \propto \begin{cases} a & \text{mat} \\ a^2 & \text{rad} \\ a^{-2} & \Lambda \end{cases}$$

Today: $\Omega$ $< 1.1$

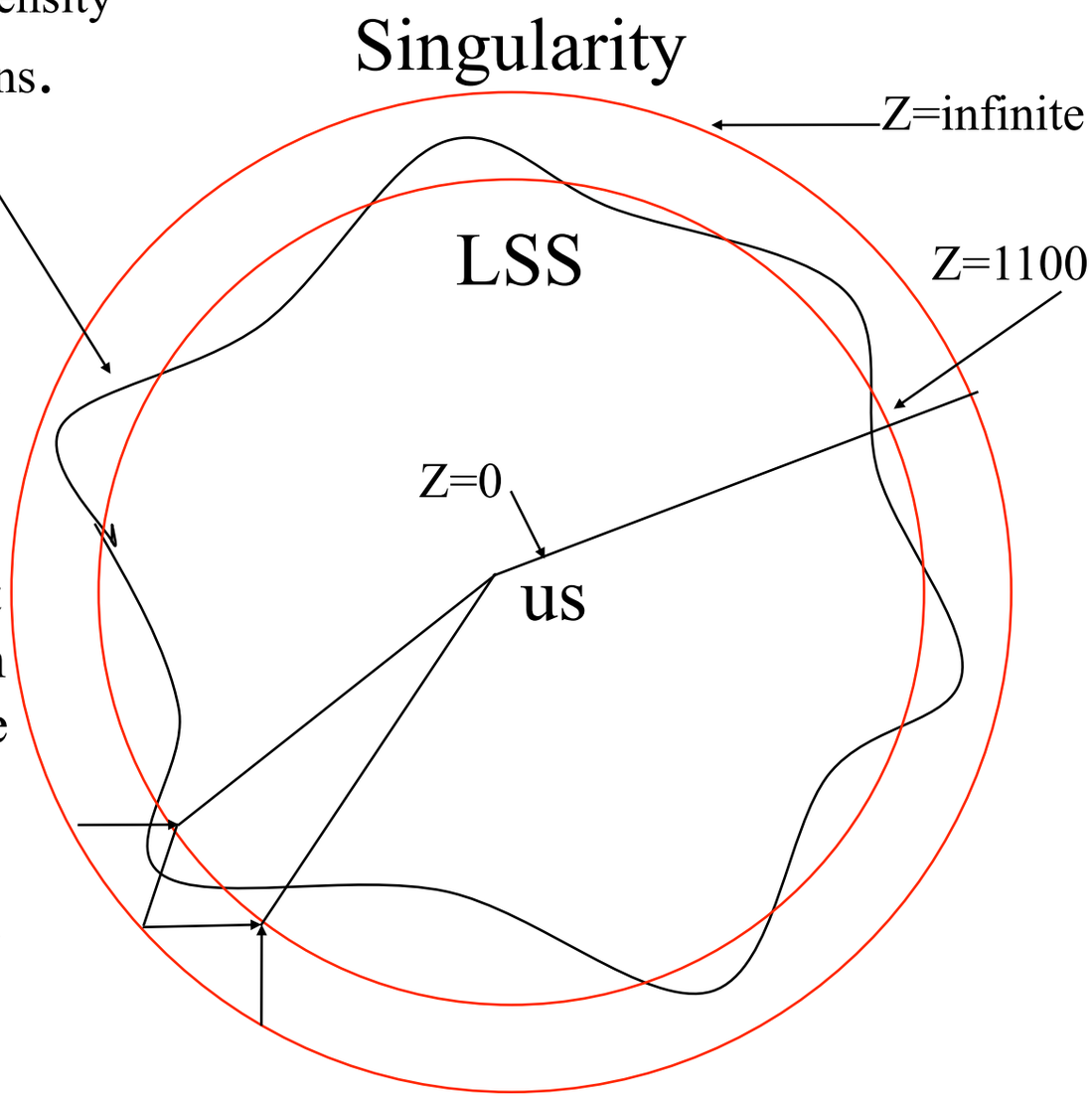
Why?

$$|\Omega(1\mathrm{s}) - 1| = O(10^{-16})$$

# Horizon problem

Primordial density fluctuations.

CMB photons emitted from opp sides of sky are in thermal equilibrium at same temp – but no time for them to interact before photons were emitted because of finite horizon size.




CMBR last interacted at 1+Z = 1100

300,000 yrs after big bang

Hubble radius was 2 degrees, 200 Mpc

LSS thickness – 15Mpc

**Any region separated by > 2 deg – causally separated at decoupling.**

# Monopole problem

Monopoles are generic prediction of GUT type models.

They are massive stable objects, like domain walls and cosmic strings and many moduli fields.

They scale like cold dark matter, so in the early universe would rapidly come to dominate the energy density.

Must find a mechanism to dilute them or avoid forming them.

# The big questions in cosmology today

a) What is dark matter? -- 23% of the energy density

b) What is dark energy? -- 73% of the energy density. Does dark energy interact with other stuff in the universe?

c) Is dark energy really a new energy form or does the accelerating universe signal a modification of our theory of gravity?

d) What is the origin of the density perturbations, giving rise to structures?

e) Is there a cosmological gravitational wave background?

f) Are the fluctuations described by Gaussian statistics? If there are deviations from Gaussianity, where do they come from?

g) How many dimensions are there? Why do we observe only three spatial dimensions?

h) Was there really a big bang (i.e. a spacetime singularity)? If not, what was there before?

# A bit of thermodynamics - remember your stat mech

Gas -weakly interacting in kinetic eqm. Distribution function for particle species x, physical momentum p

- sign bosons, + sign fermions, μ chemical pot, T-temp

$$f_x(p) = \frac{1}{e^{\frac{E_x - \mu_x}{T}} \pm 1}$$

$$E_x^2 = p^2 + m_x^2$$

Include internal dof: i.e. spin by $g_x$ (photons have g=2, neutrinos g=1)

number density:

$$n_x = \frac{g_x}{(2\pi)^3} \int f_x(p) d^3p$$

energy density:

$$\rho_x = \frac{g_x}{(2\pi)^3} \int E_x(p) f_x(p) d^3p$$

pressure:

$$p_x = \frac{g_x}{(2\pi)^3} \int \frac{|p|^2}{3E_x(p)} f_x(p) d^3p$$

Non-Rel limit : m>>T

$$n_x \simeq g_x \left(\frac{m_x T}{2\pi}\right)^{\frac{3}{2}} e^{-\frac{m_x}{T}}$$

$$\rho_x \simeq m_x n_x \qquad p_x \simeq T n_x$$

Rel limit : m<<T -- BE and FD

$$n_x^{BE} = \frac{\zeta(3)}{\pi^2} g_x T^3 \qquad n_x^{FD} \simeq \frac{3}{4} n_x^{BE}$$

$$\rho_x^{BE} \simeq \frac{\pi^2}{30} g_x T^4 \qquad \rho_x^{FD} \simeq \frac{7}{8} \rho_x^{BE}$$

$$\zeta(3) = 1.202...$$

Friedmann eqn in early universe during rad dom: $\rho_{\rm rad} = \rho_{BE} + \rho_{FD} = \frac{\pi^2}{30} g_{\rm eff}(T) T^4$

Temp high so all particle species in therm eqm: for std model particles T>1TeV.
Total num of dof for fermions (90), gauge and Higgs (28) so:

$$g_{\rm eff}(T = 1TeV) = 106.75$$

If the interaction rate between particles becomes smaller than the expansion rate, then those particles have a smaller temp than the photons (temp T) but might be relativistic. So, intro specific temp for each relativistic species.

$$g_{\rm eff}(T) = \sum_{i={\rm bosons}} g_i \left(\frac{T_i}{T}\right)^4 + \frac{7}{8} \sum_{j={\rm fermions}} g_j \left(\frac{T_j}{T}\right)^4$$

Hence: $$H = 0.33\sqrt{g_{\rm eff}}\frac{T^2}{m_{\rm Pl}} \quad \text{and} \quad t = 1.52\frac{m_{\rm Pl}}{\sqrt{g_{\rm eff}}T^2}$$

Kinetic Equilibrium - characterised by T - particles exchange energy, energy density constant:

$$X_1 + X_2 \leftrightarrow X_1 + X_2$$

Chemical Equilibrium - characterised by μ - species can change number, number density constant:

$$X_1 + X_2 \leftrightarrow X_3 + X_4 \quad \text{with} \quad \mu_1 + \mu_2 = \mu_3 + \mu_4$$

Equilibrium condition: interaction rate happens faster than the expansion rate of the universe.

$$\Gamma > H$$

Now: $\Gamma = n < \sigma v >$ — Thermal Ave

Number density; Cross section; Ave vel

Ex: Neutrino decoupling: $\gamma\gamma \leftrightarrow e^+e^- \quad \nu e \leftrightarrow \nu e \quad \nu\bar{\nu} \leftrightarrow \nu\bar{\nu}$

Cross section:

$$\sigma \simeq G_F^2 T^2 \rightarrow \Gamma \simeq G_F^2 T^5$$

$$\frac{\Gamma}{H} = \left(\frac{T}{1\text{MeV}}\right)^3$$

So for T>1 MeV, neutrinos in thermal eqm with photons, but below 1MeV, interaction rate too low to maintain eqm with photon plasma.

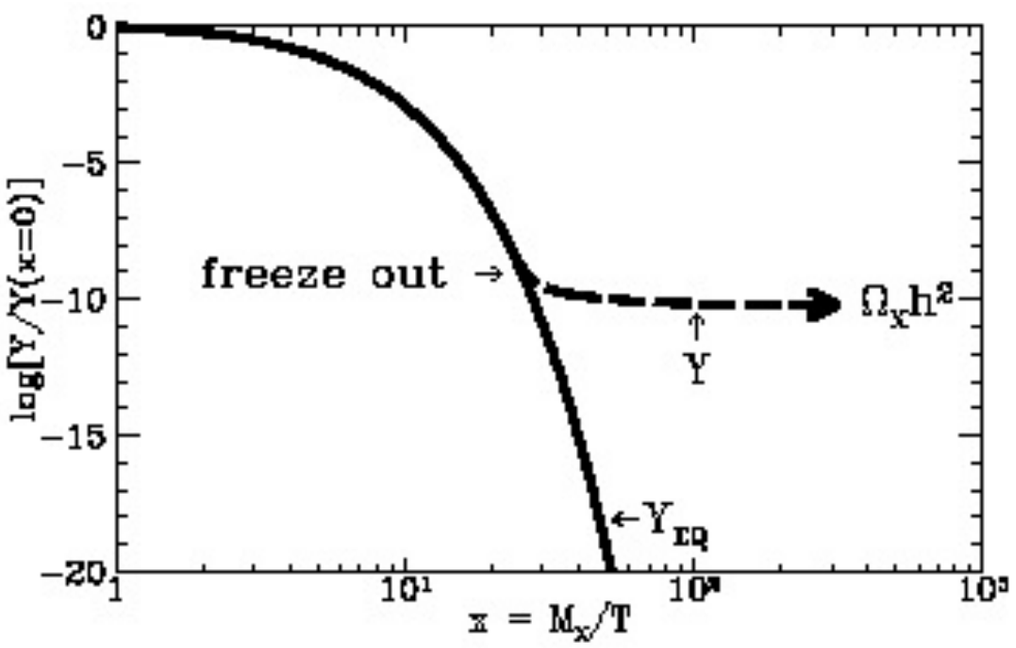
Decoupling: - departure from Kinetic Equilibrium
Freeze out: - departure from Chemical Equilibrium

Estimate decoupling or freeze out temp by Γ=H:

$$n < \sigma v > \simeq \sqrt{g_{\rm eff}} \frac{T^2}{m_{\rm Pl}}$$

Note that for neutrinos with m<1 MeV, we have m<T hence relativistic. Such particles which are relativistic at freeze-out are hot-dark-matter candidates.

Weakly interacting particles tend to have m/T ~ 20, so non-relativistic particles and cold dark matter candidates.




Taken from http://nedwww.ipac.caltech.edu/level5/Kolb/Kolb5_1.html

Y - ratio of number density to entropy density

Turns out cold dark matter needed for structure formation. Doesn't match observations if it is hot.

# Dark matter candidates:

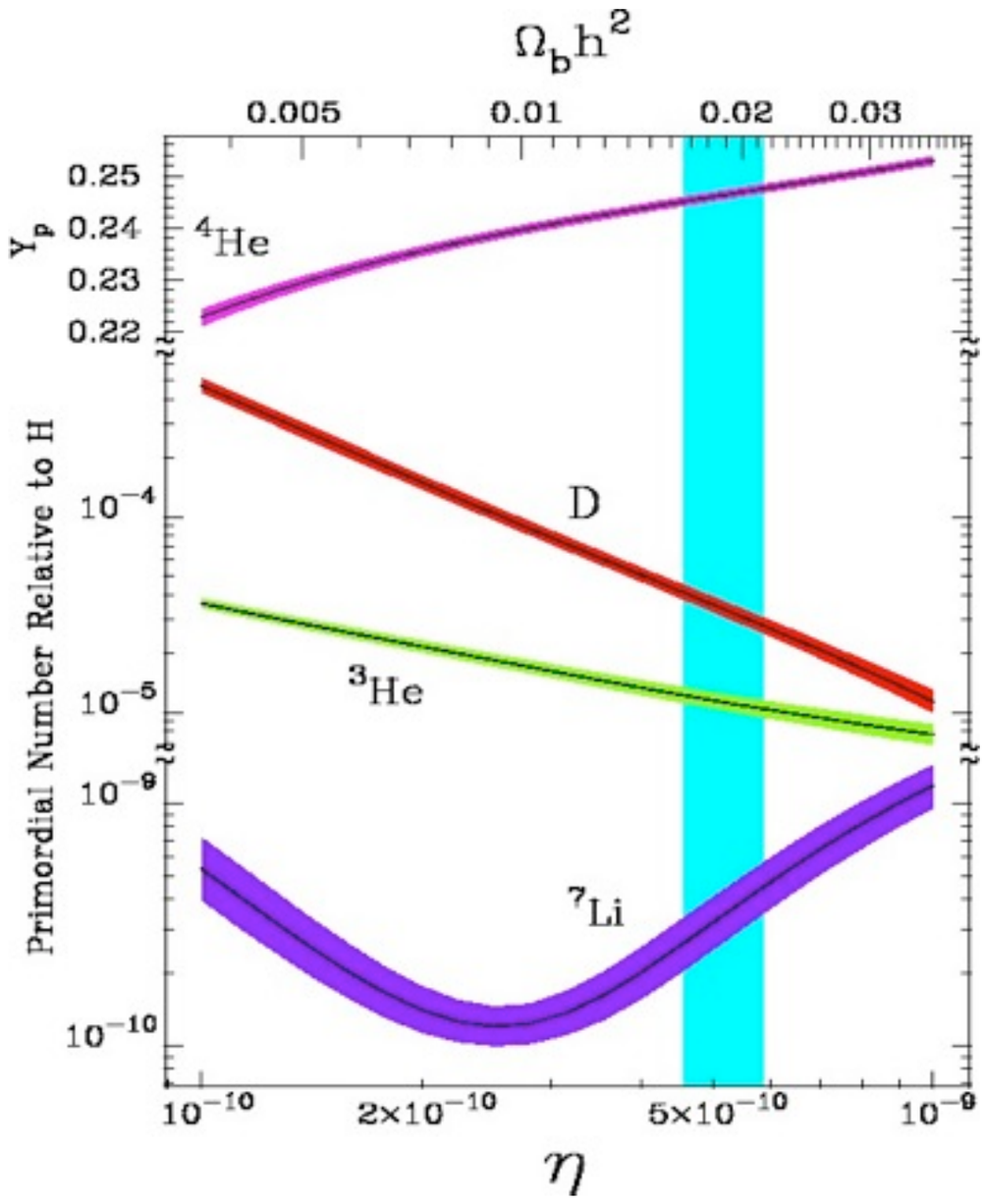
$$\Omega_m h^2 = 0.1369 \pm 0.0037$$

* Axion (solves CP problem of QCD)
* Neutrino – known to have mass, cannot be dominant dark matter.
* Neutralino – lightest supersymmetric particle.
* Gravitinos, Q-balls, WIMP-zillas…
* Kaluza-Klein dark matter
* Black holes
* …

## Big Bang Nucleosynthesis -- formation of the lightest nuclei

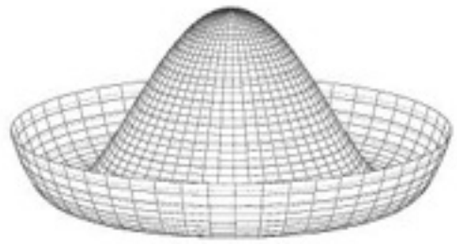
If the temperature is low enough, protons and neutrons can bind together to produce elements such as $^4$He, D, $^7$Li. For this to happen, the temperature must drop below about 1 MeV.

• Binding starts at T below the binding energy of the nuclei.
•During BBN the light elements are produced (in particular 3He, 4He, D, $^7$Li). Heavier elements are created in stars at a much later time.
•Can predict the abundances as a function of the energy density in baryons-- a great success of the Hot Big Bang




$$\Omega_b h^2 = 0.0225 \pm 0.0005 \ (68\% \ CL)$$

Phase Transitions in the Early Universe -- could be vital!
Spontaneous symmetry breaking : Higgs, topological defects, ...
Finite temp effective potential:



$$V_T(\phi) = \left(-\frac{1}{2}m^2 + \frac{\lambda}{8}T^2\right)\phi^2 + \frac{1}{4}\lambda\phi^4 + K$$

$T > \dfrac{2m}{\sqrt{\lambda}}$ then $m_{\rm eff} > 0$ and $<\phi> = 0$ symmetry restored

$T < \dfrac{2m}{\sqrt{\lambda}}$ then $m_{\rm eff} < 0$ and $<\phi> \neq 0$ symmetry broken

Example: GUT phase transition, Electroweak PT, QCD PT
Formation of topological defects such as cosmic strings, domain walls, monopoles, textures ...

I owe a great deal to cosmic strings -- they are neat and through cosmic superstrings could provide the first observational evidence for string theory.

# Invisibles 2013
# Cosmology - Lecture 2

**Ed Copeland -- Nottingham University**

Dark Energy - Dark Matter - Modified Gravity

# Weighing the Universe

$1.\Omega_m$

a. Cluster baryon abundance using X-ray measurements of intracluster gas, or SZ measurements.

b. Weak grav lensing and large scale peculiar velocities.

c. Large scale structure distribution.

d. Numerical simulations of cluster formation.

$$\Omega_m h^2 = 0.1369 \pm 0.0037$$

**(Komatsu et al, 2008) (WMAP5)**

$$\Omega_m << 1$$

**$H_0$=70.4±1.3 km $s^{-1}$ $Mpc^{-1}$**

$2.\Omega_b$ **BBN** $\Omega_b h^2 = 0.0225 \pm 0.0005 \ (68\% \ CL)$

Majority of baryonic matter dark.

$\Omega_b << \Omega_m$ → **Require Dark matter !!**

**Candidates: WIMPS (Neutralinos, Kaluza Klein Particles, Universal Extra Dimensions...)**

**Axinos, Axions, Axion-like light bosons, Sterile neutrinos, Q-balls, WIMPzillas, Elementary Black Holes...**

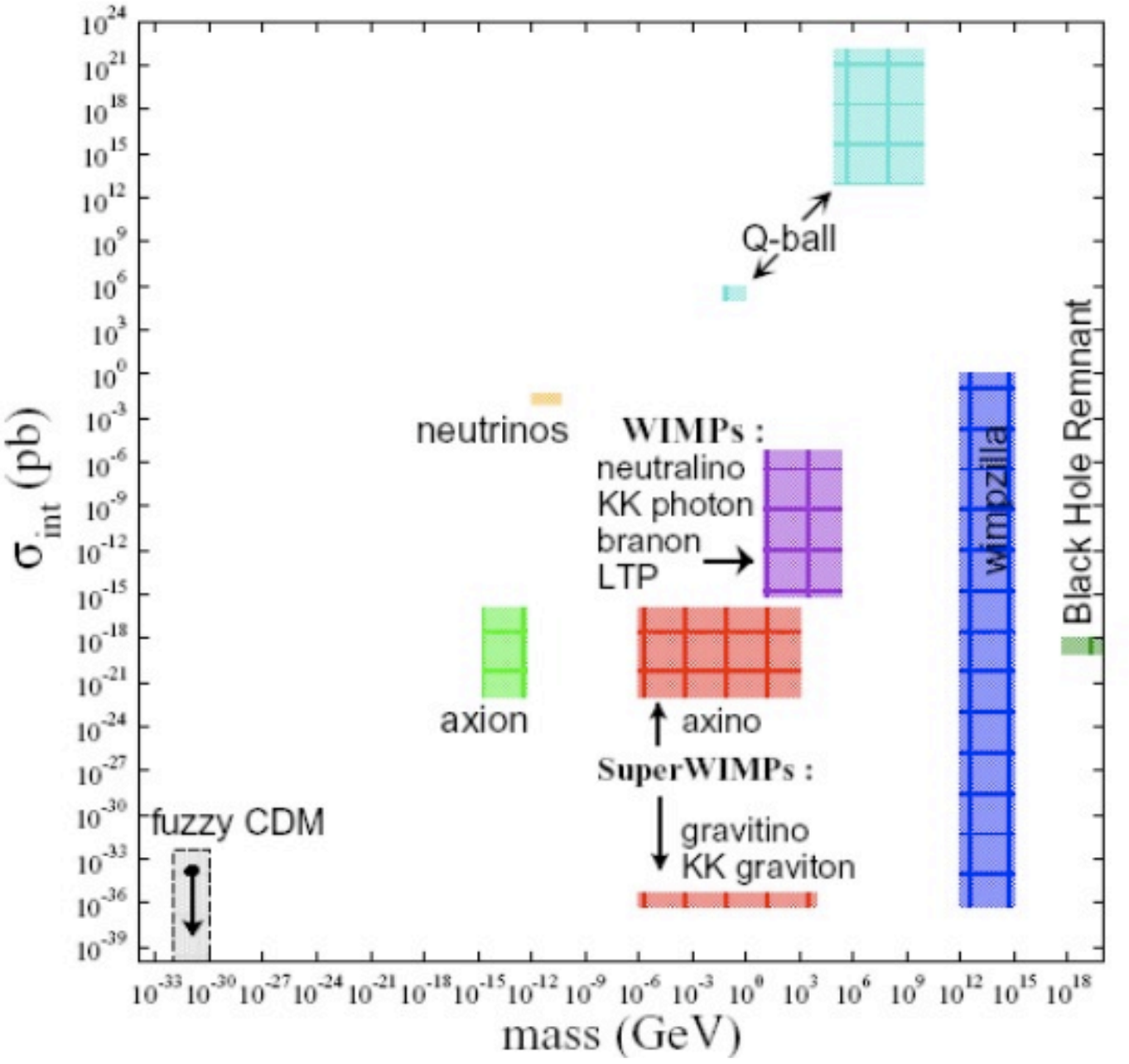
**Search for them is on:**

**1. Direct detection -- 20 expts worldwide**

**2. Indirect detection -- i.e. Bullet Cluster !**

**3. LHC -- i.e. missing momentum and energy**

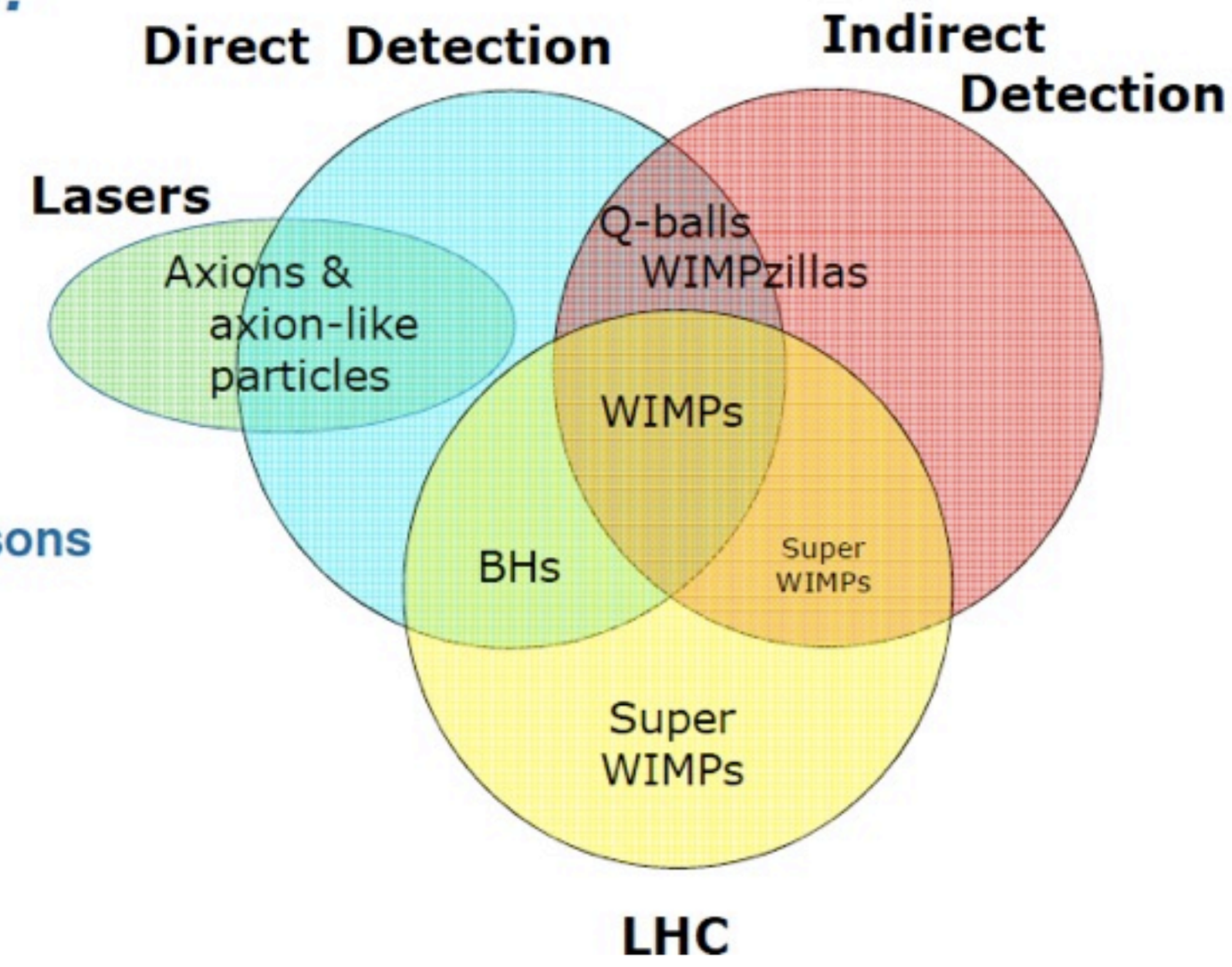
# Dark Matter Candidates




C. Spiering, Cosmo 09

## *DM candidates:*

- **WIMPs**
  - Neutralinos
  - Kaluza-Klein particles
  - …
- **Axinos**
- **Super-WIMPs**
- **Axions**
- **Axion-like light bosons**
- **Sterile neutrinos**
- **Q-balls**
- **WIMPzillas**
- **Elementary BHs**
- **…**



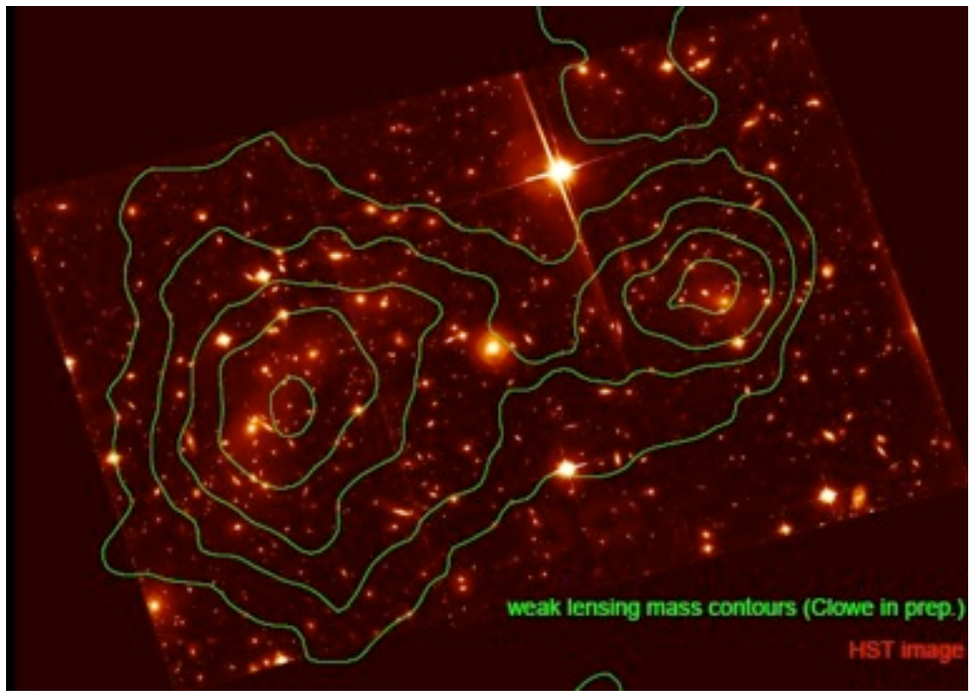

**C. Spiering, Cosmo 09**

**Indirect evidence for Dark Matter -- Bullet Cluster**

**Two clusters of galaxies colliding.**

**Dark matter in each passes straight through and doesn't interact -- seen through weak lensing in right image.**

**Ordinary matter in each interacts in collision and heats up -- seen through infra red image on left.**




**Clowe et al 2006**

## Evidence for Dark Energy?

### Enter CMBR:

$$3.\Omega_0 = \Omega_m + \Omega_\Lambda$$

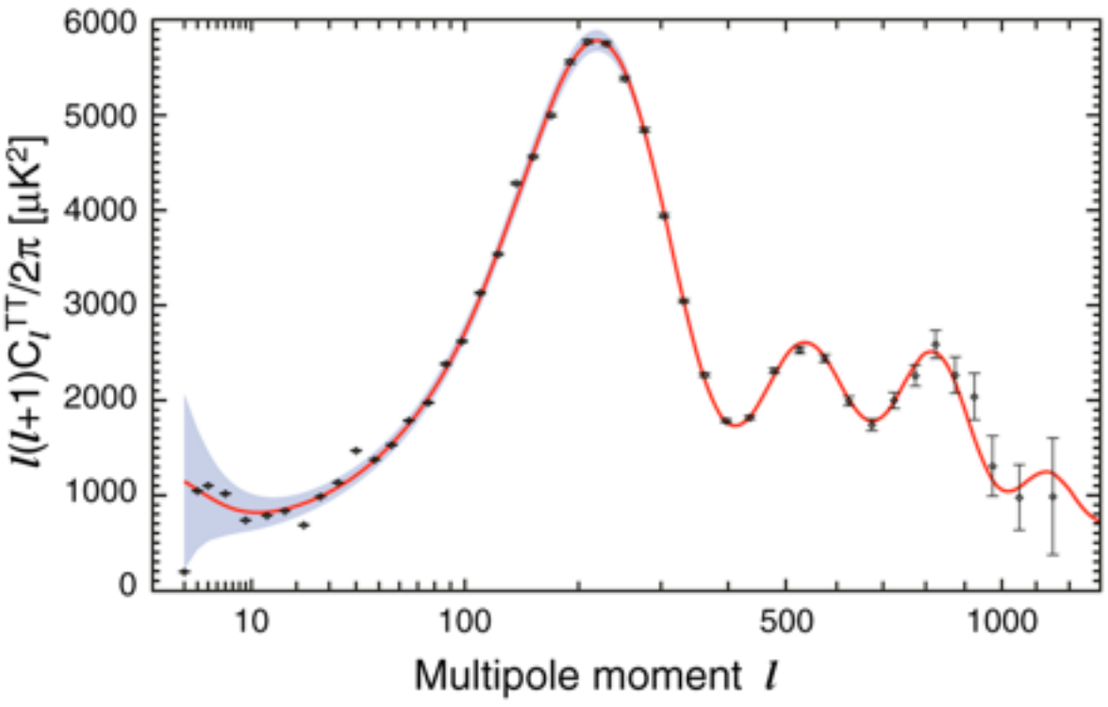
**Provides clue. 1st angular peak in power spectrum.**

$$l_{peak} \approx \frac{220}{\sqrt{\Omega_0}}$$

$$\left|1-\Omega_0\right| = 0.03^{+0.026}_{-0.025}$$

**WMAP3-Depends on assumed priors**

**Spergel et al 2006**




$$-0.0175 < \Omega_k < 0.0085$$

**Dunkley et al 2008 (WMAP5)**

# WMAP7 and dark energy

**(Komatsu et al, 2010)**

Assume flat univ + +BAO+ SNLS:

$$w = -0.980 \pm 0.053$$

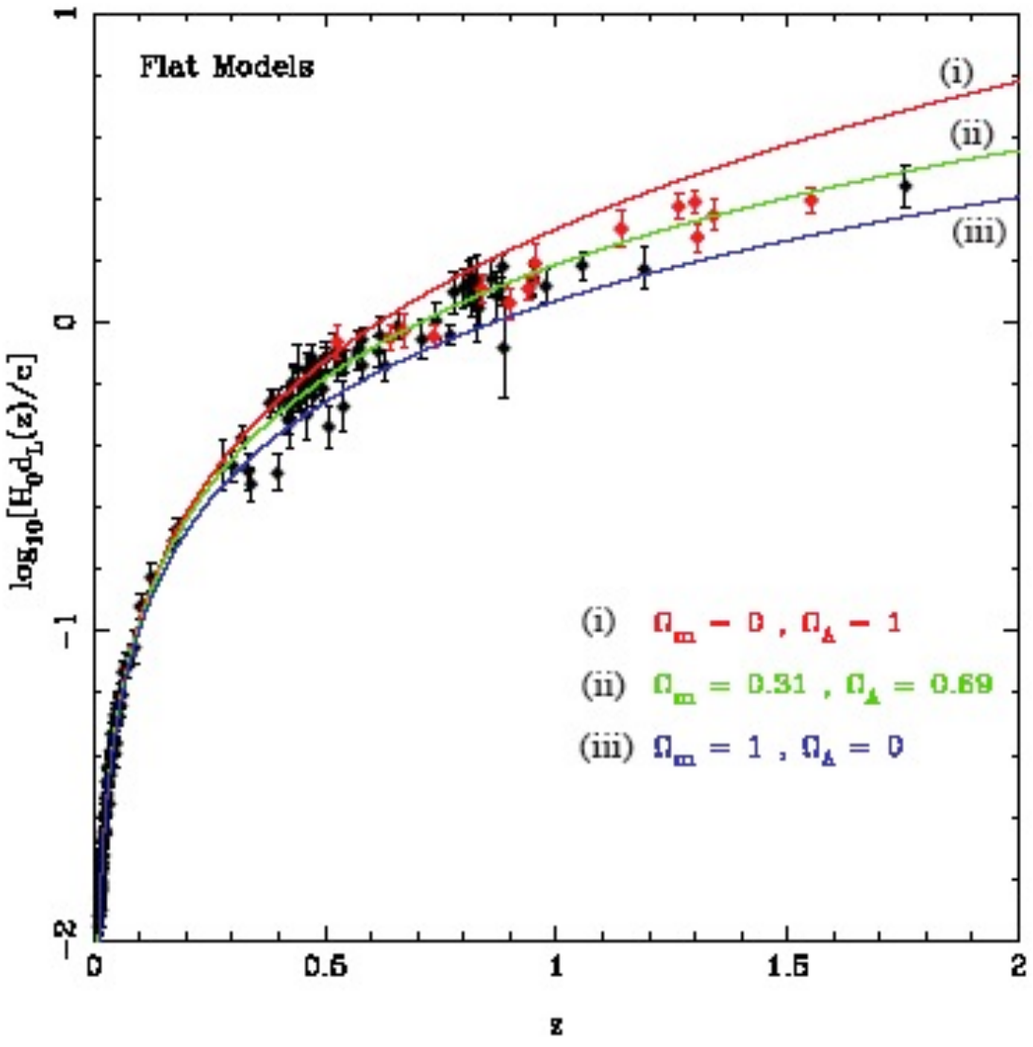
Drop prior of flat univ: WMAP + BAO + SNLS:

$$w = -0.999^{+0.057}_{-0.056} \quad \Omega_k = -0.0057^{+0.0067}_{-0.0068}$$

Drop assumption of const w but keep flat univ: WMAP + BAO + SNLS:

$$\begin{aligned} w_0 &= -0.93 \pm 0.12 \\ w_a &= -0.38^{+0.66}_{-0.65} \end{aligned}$$

# Type Ia Luminosity distance v z [Reiss et al 2004]




**Flat model**
**Black dots -- Gold data set**
**Red dots -- HST**

$(i)\Omega_m = 0,\ \Omega_\Lambda = 1 \quad (ii)\Omega_m = 0.31,\ \Omega_\Lambda = 0.69 \quad (iii)\Omega_m = 1,\ \Omega_\Lambda = 0$

## Coincidence problem – why now?

Recall: $\frac{\ddot{a}}{a} \geq 0 < - > = (\rho + 3p) \leq 0$

If: $\rho_x = \rho_x^0 \, a^{-3(1+w_x)}$

**Universe dom by dark energy at:** $z_x = \left(\frac{\Omega_x}{\Omega_m}\right)^{\frac{1}{3w_x}} - 1$

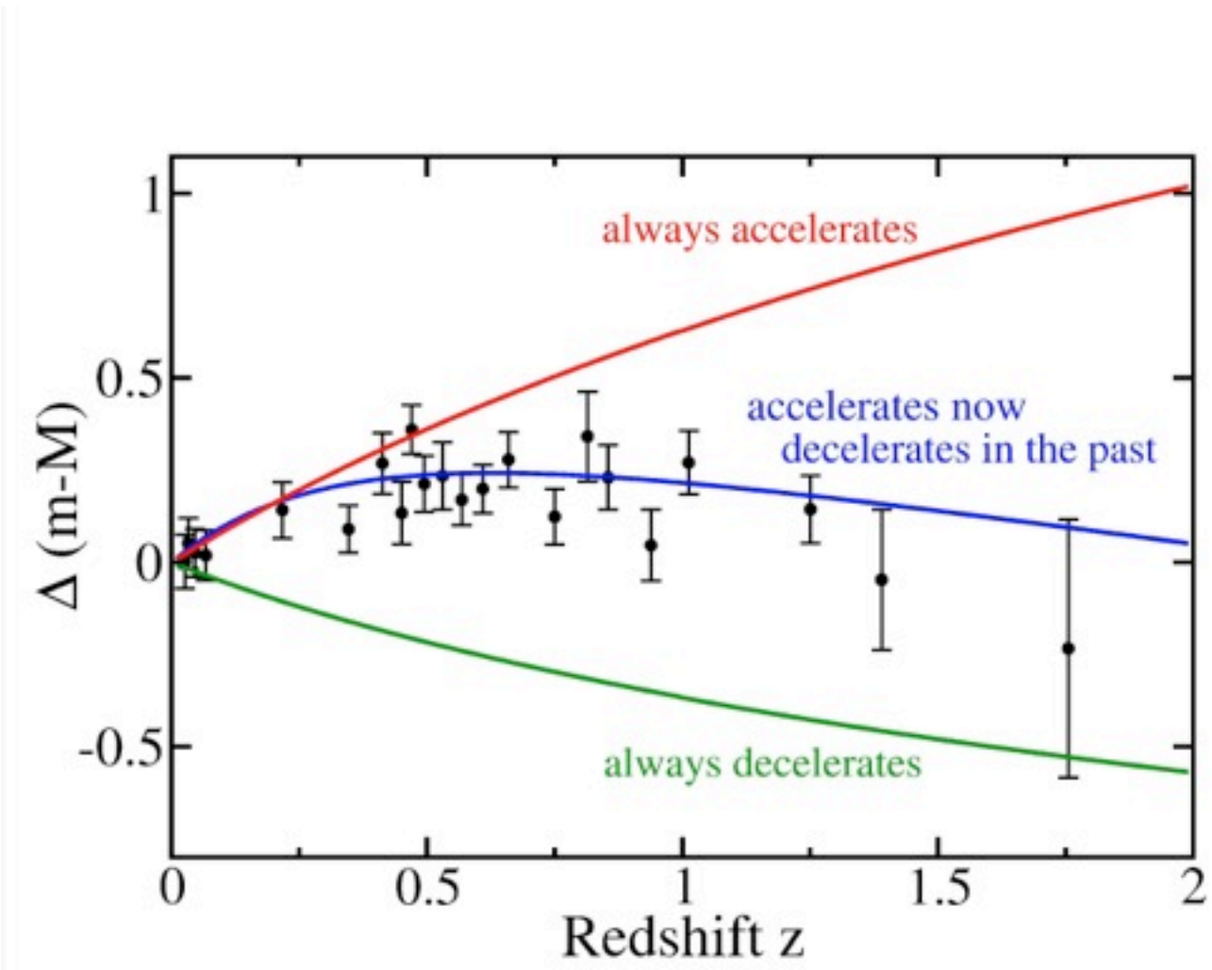
$$\left(\frac{\Omega_x}{\Omega_m}\right) = \frac{7}{3} \rightarrow z_x = 0.5,\ 0.3 \text{ for } w_x = -\frac{2}{3},\ -1$$

**Univ accelerates at:** $z_a = \left(-(1+3w_x)\frac{\Omega_x}{\Omega_m}\right)^{\frac{-1}{3w_x}} - 1$

$$z_a = 0.7,\ 0.5 \text{ for } w_x = -\frac{2}{3},\ -1$$

Constraint: $-0.11 < 1 + w < 0.14$ **Komatsu et al 2008 (WMAP5)**

The acceleration has not been forever -- pinning down the turnover will provide a very useful piece of information.



**What is making the Universe accelerate?**

**Dark energy -- a weird form of energy that exists in empty space and pervades the universe -- also known as vacuum energy or cosmological constant.**

**Smoothly distributed, doesn't cluster.**

**Constant density or very slowly varying**

**Doesn't interact with ordinary matter -- only with gravity**

**Big problem though. When you estimate how much you expect there to be, from the Quantum world, the observed amount is far less than expected.**

**Theoretical prediction = $10^{120}$ times observation**

# The problem with the cosmological constant

$$R_{\mu\nu} - \tfrac{1}{2}Rg_{\mu\nu} - \lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Einstein (1917) -- static universe with dust

Not easy to get rid of it, once universe found to be expanding.

Anything that contributes to energy density of vacuum acts like a cosmological constant

$$< T_{\mu\nu} > = < \rho > g_{\mu\nu}$$

Lorentz inv

$$\lambda_{eff} = \lambda + 8\pi G < \rho >$$

or

$$\rho_V = \lambda_{eff}/8\pi G$$

Effective cosm const

Effective vac energy

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - < \rho > | \leq \frac{3H_0^2}{8\pi G}$$

Age

Flat

Non-vac matter

$$H^2 \equiv \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho + \lambda - \frac{k}{a^2}$$

$$H_0 \simeq 10^{-10} yr^{-1} : \frac{|k|}{a_0^2} \leq H_0^2 : |\rho - <\rho>| \leq \frac{3H_0^2}{8\pi G}$$

Hence: $\lambda_{eff} \leq H_0^2 \; or |\rho_V| \leq 10^{-29} g cm^{-3} \simeq 10^{-47} GeV^4$

Problem: expect <ρ> of empty space to be much larger. Consider summing zero-point energies (ħω/2) of all normal modes of some field of mass m up to wave number cut off Λ>>m:

$$<\rho> = \int_0^{\Lambda} \frac{4\pi k^2 dk}{2(2\pi)^3} \sqrt{k^2 + m^2} \simeq \frac{\Lambda^4}{16\pi^2}$$

For many fields (i.e. leptons, quarks, gauge fields etc...):

$$<\rho> = \frac{1}{2} \sum_{\text{fields}} g_i \int_0^{\Lambda_i} \sqrt{k^2 + m^2} \frac{d^3k}{(2\pi)^3} \simeq \sum_{\text{fields}} \frac{g_i \Lambda_i^4}{16\pi^2}$$

where $g_i$ are the dof of the field (+ for bosons, - for fermions).

Imagine just one field contributed an energy density $\rho_{cr} \sim (10^{-3}\ \text{eV})^4$. Implies the cut-off scale Λ<0.01 eV -- well below scales we understand the physics of.

Planck scale: $\Lambda \simeq (8\pi G)^{-1/2} \rightarrow <\rho> \simeq 2 \times 10^{71} GeV^4$

But: $|\rho_V| = |<\rho> + \lambda/8\pi G| \leq 2 \times 10^{-47} GeV^4$

Must cancel to better than 118 decimal places.

Even at QCD scale require 41 decimal places!

Very unlikely a classical contribution to the vacuum energy density will cancel this quantum contribution to such high precision

Not all is lost -- what if there is a symmetry present to reduce it? Supersymmetry does that. Every boson has an equal mass SUSY fermion partner and vice-versa, so their contributions to $<\rho>$ cancel.

However, SUSY seems broken today - no SUSY partners have been observed, so they must be much heavier than their standard model partners. If SUSY broken at scale M, expect $<\rho> \sim M^4$ because of breakdown of cancellations. Current bounds suggest M~1TeV which leads to a discrepancy of 60 orders of magnitude as opposed to 118 !

Still a problem of course -- is there some unknown mechanism perhaps from quantum gravity that will make the vacuum energy vanish ?

# Different approaches to Dark Energy include amongst many:

- A true cosmological constant -- but why this value?
- Solid –dark energy such as arising from frustrated network of domain walls.
- Time dependent solutions arising out of evolving scalar fields -- Quintessence/K-essence.
- Modifications of Einstein gravity leading to acceleration today.
- Anthropic arguments.
- Perhaps GR but Universe is inhomogeneous.

# Early evidence for a cosmological constant type term.

1987: Weinberg argued that anthropically $\rho_{vac}$ could not be too large and positive otherwise galaxies and stars would not form. It should not be very different from the mean of the values suitable for life which is positive, and he obtained $\Omega_{vac} \sim 0.6$

1990: Observations of LSS begin to kick in showing the standard $\Omega_{CDM}$ =1 struggling to fit clustering data on large scales, first through IRAS survey then through APM (Efstathiou et al).

1990: Efstathiou, Sutherland and Maddox - Nature (238) -- explicitly suggest a cosmology dominated today by a cosmological constant with $\Omega_{vac} < 0.8$ !

1998: Type Ia SN show striking evidence of cosm const and the field takes off.

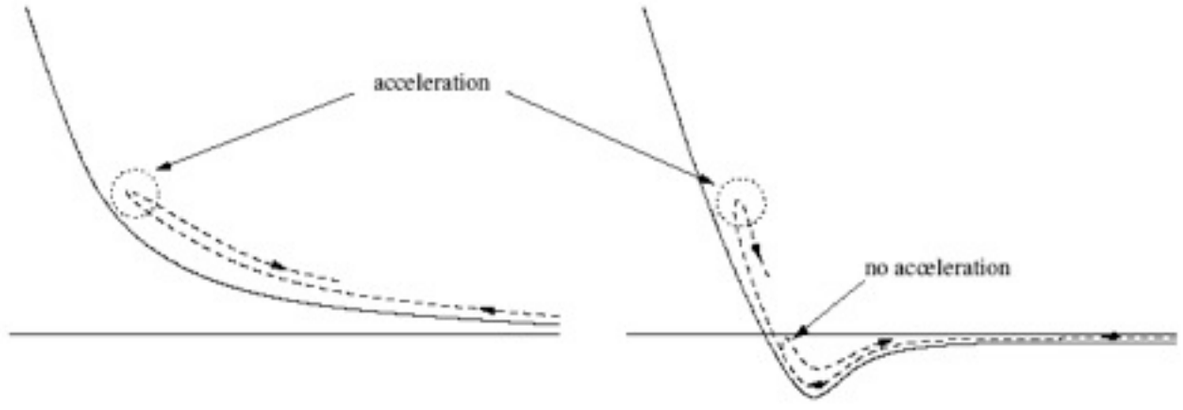
# String/M-theory -- where are the realistic models?

`No go' theorem: forbids cosmic acceleration in cosmological solutions arising from compactification of pure SUGR models where internal space is time-independent, non-singular compact manifold without boundary --[Gibbons]

Recent extension: forbids four dimensional cosmic acceleration in cosmological solutions arising from warped dimensional reduction --[Wesley 08]

## Avoid no-go theorem by relaxing conditions of the theorem.

1. Allow internal space to be time-dependent, analogue of time-dependent scalar fields (radion)




Current realistic potentials are too steep

Models kinetic, not matter domination before entering

accelerated phase.

Four form Flux and the cosm const: [Bousso and Polchinski]

Effective 4D theory from $M^4 x S^7$ compactification

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\frac{1}{2\kappa^2}R + \Lambda_b - \frac{1}{2\cdot 4!}F_4^2\right)$$

Negative bare cosm const: $-\Lambda_b$

EOM: $\nabla_\mu(\sqrt{-g}F^{\mu\nu\rho\sigma}) = 0 \longrightarrow F^{\mu\nu\rho\sigma} = c\epsilon^{\mu\nu\rho\sigma}$

Eff cosm const: $\Lambda = -\Lambda_b - \frac{1}{48}F_4^2 = -\Lambda_b + \frac{c^2}{2}$

Quantising c and considering J fluxes $\Lambda = -\Lambda_b + \frac{1}{2}\sum_{i=1}^{J} n_i^2 q_i^2$

Observed cosm const with J~100

Still needed to stabilise moduli but opened up way of obtaining many de Sitter vacua using fluxes -- String Landscape in which all the vacua would be explored because of eternal inflation.

# 1.The String Landscape approach

Type IIB String theory compactified from 10 dimensions to 4.

Internal dimensions stabilised by fluxes.

Many many vacua ~ $10^{500}$ !

Typical separation ~ $10^{-500}$ $\Lambda_{pl}$

Assume randomly distributed, tunneling allowed between vacua --> separate universes .

Anthropic : Galaxies require vacua < $10^{-118}$ $\Lambda_{pl}$ [Weinberg] Most likely to find values not equal to zero!

[Witten 2008]

## Landscape gives a realisation of the multiverse picture.

There isn't one true vacuum but many so that makes it almost impossible to find our vacuum in such a Universe which is really a multiverse.

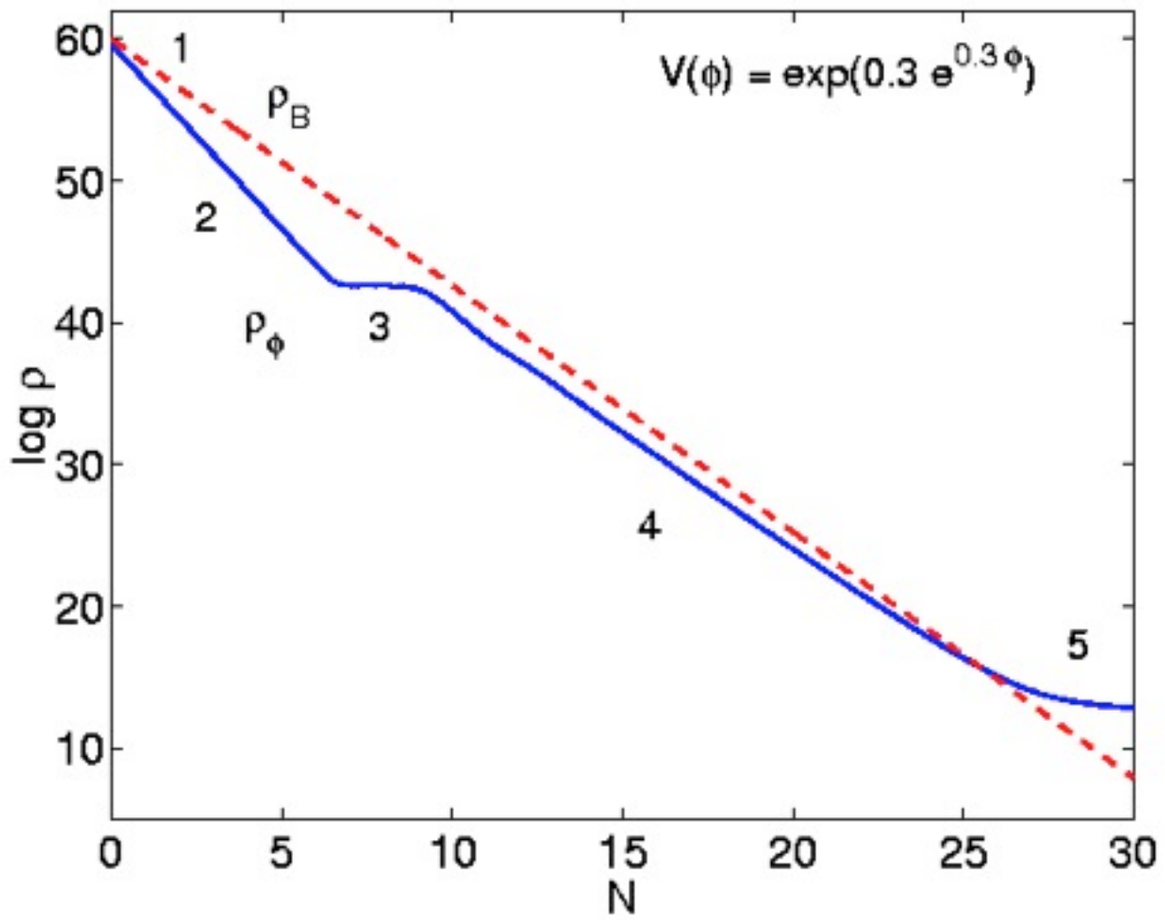
So how can we hope to understand or predict why we have our particular particle content and couplings when there are so many choices in different parts of the universe, none of them special ?

This sounds like bad news, we will rely on anthropic arguments to explain it through introducing the correct measures and establishing peaks in probability distributions.

Or perhaps, it isn't a cosmological constant, but a new field such as Quintessence which will eventually drive us to a unique vacuum with zero vacuum energy -- that too has problems, such as fifth force constraints, as we will see.

# Slowly rolling scalar fields
# Quintessence - Generic behaviour

1. **PE → KE**
2. **KE dom scalar field energy den.**
3. **Const field.**
4. **Attractor solution: almost const ratio KE/ PE.**
5. **PE dom.**



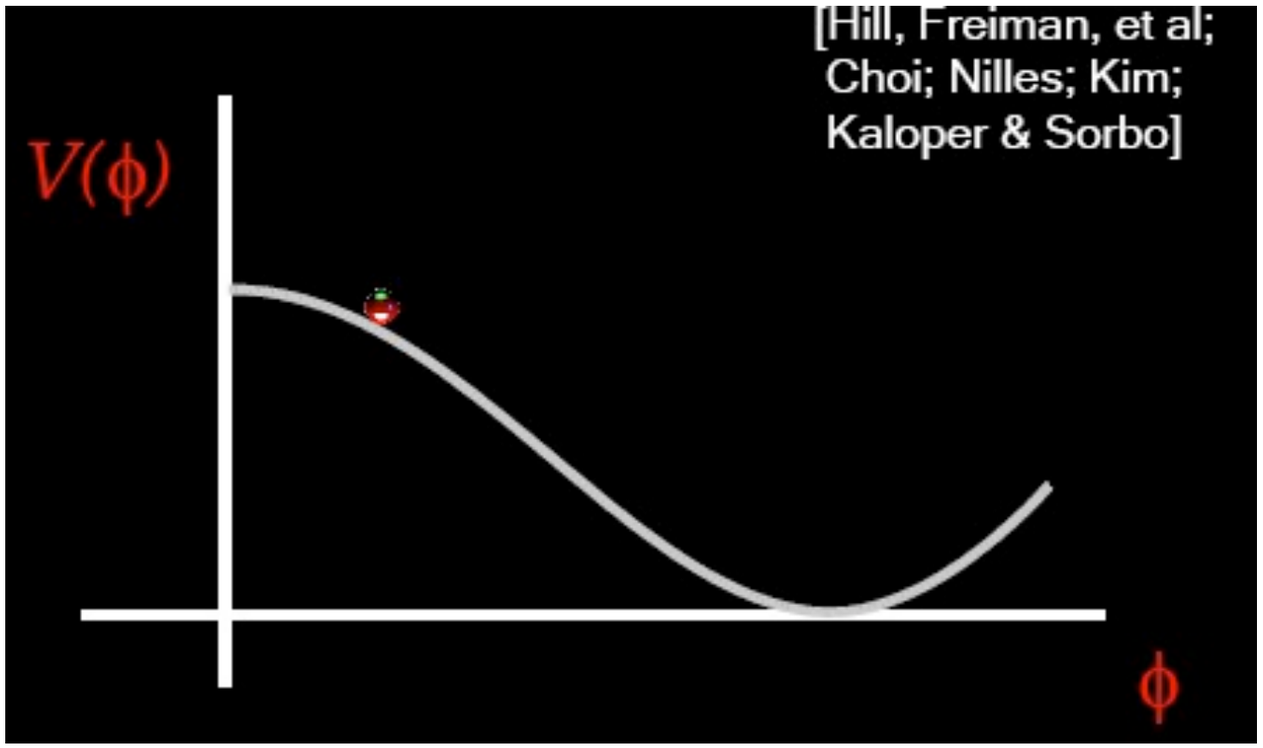

Nunes

**Attractors make initial conditions less important**

**Particle physics inspired models?**

**Pseudo-Goldstone Bosons -- approx sym $\phi$ --> $\phi$ + const.**

**Leads to naturally small masses, naturally small couplings**




Barbieri et al

$$V(\phi) = \lambda^4(1 + \cos(\phi/F_a))$$

Axions could be useful for strong CP problem, dark matter and dark energy.

### 1. Chameleon fields [Khoury and Weltman (2003) …]

Key idea: in order to avoid fifth force type constraints on Quintessence models, have a situation where the mass of the field depends on the local matter density, so it is massive in high density regions and light (m~H) in low density regions (cosmological scales).

### 2. Phantom fields [Caldwell (2002) …]

The data does not rule out w<-1. Can not accommodate in standard quintessence models but can by allowing negative kinetic energy for scalar field (amongst other approaches).

### 3. K-essence [Armendariz-Picon et al …]

Scalar fields with non-canonical kinetic terms. Advantage over Quintessence through solving the coincidence model?

Long period of perfect tracking, followed by domination of dark energy triggered by transition to matter domination -- an epoch during which structures can form. Similar fine tuning to Quintessence.

# 4. Interacting Dark Energy [Kodama & Sasaki (1985), Wetterich (1995), Amendola (2000) + many others… ]

Idea: why not directly couple dark energy and dark matter?

$$\text{Ein eqn} \quad : \quad G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\text{General covariance} \quad : \quad \nabla_\mu G^\mu_\nu = 0 \rightarrow \nabla_\mu T^\mu_\nu = 0$$

$$T_{\mu\nu} = \sum_i T^{(i)}_{\mu\nu} \rightarrow \nabla_\mu T^{\mu(i)}_\nu = -\nabla_\mu T^{\mu(j)}_\nu \quad \text{is ok}$$

Couple dark energy and dark matter fluid in form:

$$\nabla_\mu T^{\mu(\phi)}_\nu = \sqrt{\frac{2}{3}}\kappa\beta(\phi) T^{\alpha(m)}_\alpha \nabla_\nu \phi$$

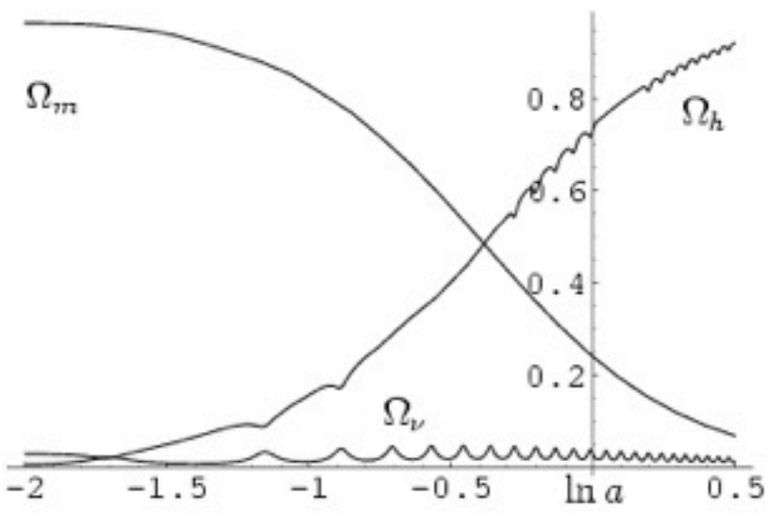
$$\nabla_\mu T^{\mu(m)}_\nu = -\sqrt{\frac{2}{3}}\kappa\beta(\phi) T^{\alpha(m)}_\alpha \nabla_\nu \phi$$

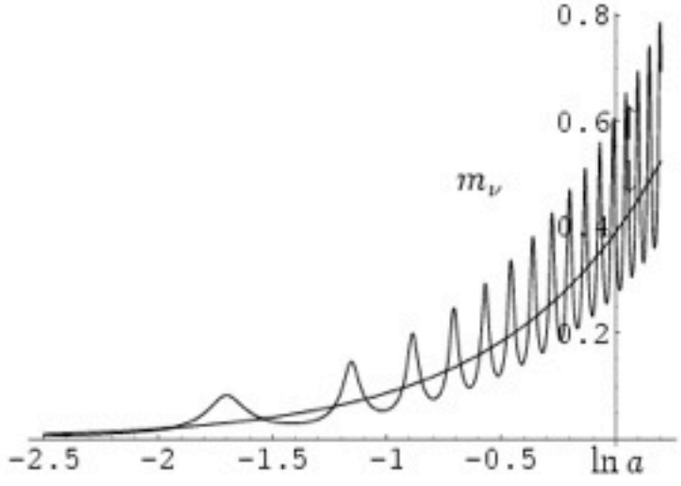
Including neutrinos -- 2 distinct DM families -- resolve coincidence problem [Amendola et al (2007)]

Depending on the coupling, find that the neutrino mass grows at late times and this triggers a transition to almost static dark energy.

Trigger scale set by when neutrinos become non-rel

$$[\rho_h(t_0)]^{\frac{1}{4}} = 1.07 \left(\frac{\gamma m_\nu(t_0)}{eV}\right)^{\frac{1}{4}} 10^{-3} eV \qquad w_0 \approx -1 + \frac{m_\nu(t_0)}{12\text{eV}}$$



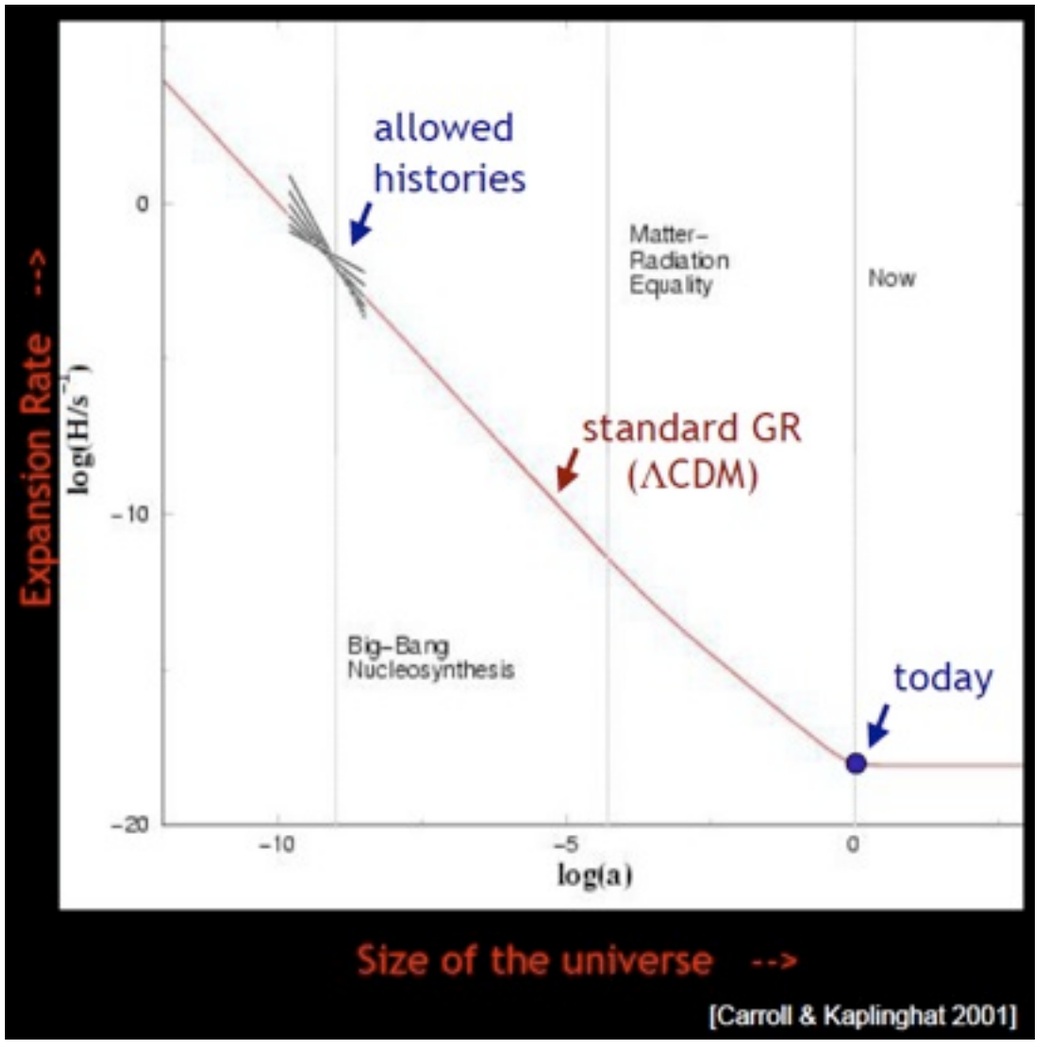

$m_\nu$

Perhaps we are wrong -- maybe the question should be not whether dark energy exists, rather should we be modifying gravity?

Has become a big industry but it turns out to be hard to do too much to General Relativity without falling foul of data.

BBN occurred when the universe was about one minute old, about one billionth its current size. It fits well with GR and provides a test for it in the early universe.

Any alternative had better deliver the same successes not deviate too much at early times, but turn on at late times .



Any theory deviating from GR must do so at late times yet remain consistent with Solar System tests. Potential examples include:

- f(R) gravity -- coupled to higher curv terms, changes the dynamical equations for the spacetime metric.

[Starobinski 1980, Carroll et al 2003, ...]

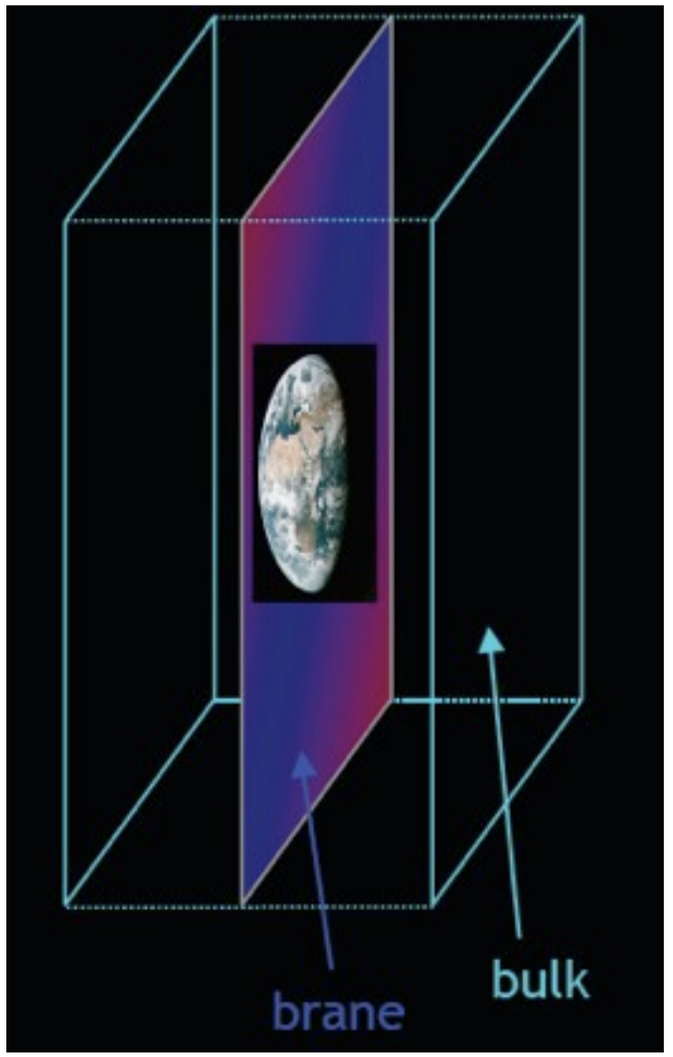
- Modified source gravity -- gravity depends on nonlinear function of the energy.

- Gravity based on the existence of extra dimensions -- DGP gravity

We live on a brane in an infinite extra dimension. Gravity is stronger in the bulk, and therefore wants to stick close to the brane -- looks locally four-dimensional.

Tightly constrained -- both from theory and observations -- ghosts !

Example of Galileon fields -- [Nicolis et al 08]




[Carroll]

Acc$^{n}$ from new Gravitational Physics? [Starobinski 1980, Carroll et al 2003, ...]

$$S = \frac{M_P^2}{2}\int d^4x\sqrt{-g}\left(R - \frac{\mu^4}{R}\right) + \int d^4x\sqrt{-g}\,\mathcal{L}_M$$

Modify Einstein

Const curv vac solutions:

$$\nabla_\mu R = 0, \rightarrow R = \pm\sqrt{3}\mu^2$$

de Sitter or Anti de Sitter

Transform to EH action:

$$\tilde{g}_{\mu\nu} = p(\phi)g_{\mu\nu}\ , \quad p \equiv \exp\left(\sqrt{\frac{2}{3}}\frac{\phi}{M_P}\right) \equiv 1 + \frac{\mu^4}{R^2}$$

Scalar field minimally coupled to gravity and non minimally coupled to matter fields with potential:

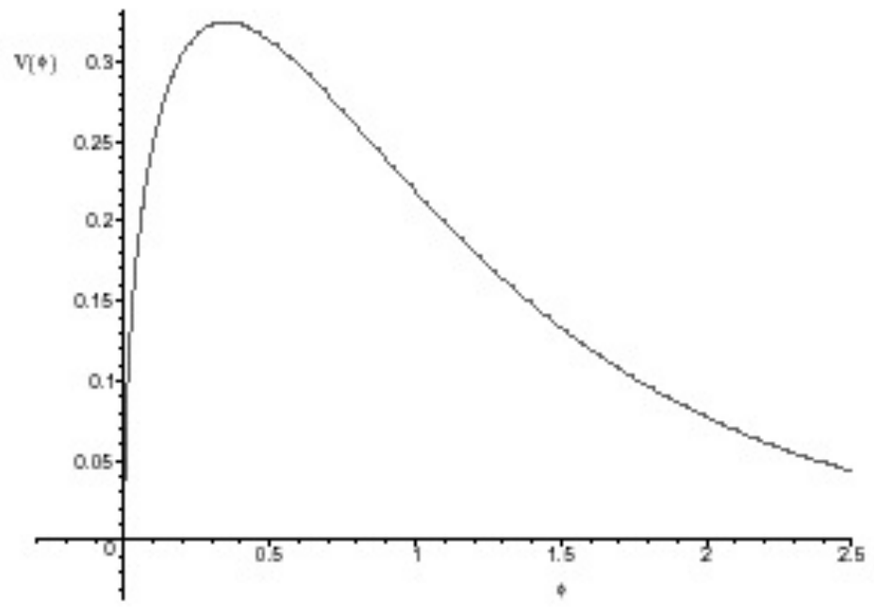
$$V(\phi) = \mu^2 M_P^2 \frac{\sqrt{p-1}}{p^2}$$

# Cosmological solutions:

**Eternal de Sitter - $\phi$ just reaches $V_{max}$ and stays there. Fine tuned and unstable.**

**Power law inflation -- $\phi$ overshoots $V_{max}$, universe asymptotes with $w_{DE}$=-2/3.**

**Future singularity-- $\phi$ doesn't reach $V_{max}$, and evolves back towards $\phi$=0.**




1.Fine tuning needed so acceleration only recently: $\mu \sim 10^{-33}$eV

2. Also, not consistent with classic solar system tests of gravity.

3. Claim that such $R^{-n}$ corrections fail to produce matter dom era [Amendola et al, 06]

But recent results based on singular perturbation theory suggests it is possible [Evans et al, 07 -- see also Carloni et al 04]

# More general f (R) models [Gurovich & Starobinsky (79); Tkachev (92); Carloni et al (04,07,09); Amendola & Tsujikawa 08; Bean et al 07; Wu & Sawicki 07; Appleby & Battye (07) and (08); Starobinsky (07); Evans et al (07); Frolov (08)… ]

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[\frac{R + f(R)}{2\kappa^2} + \mathcal{L}_\mathrm{m}\right]$$ No $\Lambda$

Usually f (R) struggles to satisfy both solar system bounds on deviations from GR and late time acceleration. It brings in extra light degree of freedom --> fifth force constraints.

Ans: Make scalar dof massive in high density solar vicinity and hidden from solar system tests by chameleon mechanism.

Requires form for f (R) where mass of scalar is large and positive at high curvature.

Issue over high freq oscillations in R and singularity in finite past.

In fact has to look like a standard cosmological constant [Song et al, Amendola et al]

To test GR on cosmological scales compare kinematic probes of dark energy to dynamical ones and look for consistency.

Kinematic probes:  only sensitive to a(t) such as standard candles, baryon oscillations.

Dynamical probes: sensitive to a(t) and structure growth such as weak lensing and cluster counts.

Determining the best way to test for dark energy and parameterise the dark energy equation of state is a difficult task, not least given the number of approaches that exist to modeling it .

Dark Energy Task Force review: Albrecht et al : astro-ph/0609591

Findings on best figure of merit: Albrecht et al: arXiv:0901.0721

# Invisibles 2013
# Cosmology - Lecture 3

**Ed Copeland -- Nottingham University**

Origin of  Inflation and the primordial density fluctuations.

# Return to the beginning -- Inflation

A period of accelerated expansion in the early Universe

Small smooth and coherent patch of Universe size less than (1/H) grows to size greater than comoving volume that becomes entire observable Universe today.

Explains the homogeneity and spatial flatness of the Universe

and also explains why no massive relic particles predicted in say GUT theories

Leading way to explain observed inhomogeneities in the Universe

$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G(\rho + 3p) \quad --- \; Accn$$

$$If \;\; \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

# What is Inflation?

Any epoch of the Universe's evolution during which the comoving Hubble length is decreasing. It corresponds to any epoch during which the Universe has accelerated expansion.

$$\frac{d}{dt}\left(\frac{H^{-1}}{a}\right) < 0 \leftrightarrow \ddot{a} > 0$$

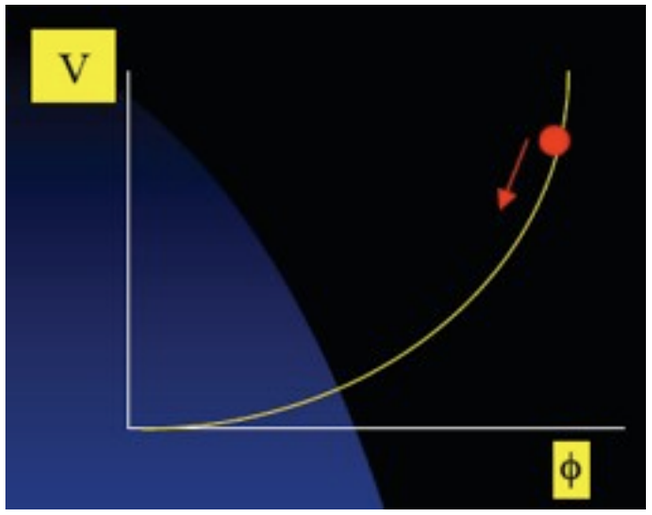
$$\frac{\ddot{a}}{a} = -\frac{8\pi}{3} G\,(\rho + 3p) - - - Accn$$

$$If\ \ \rho + 3p < 0 \Rightarrow \ddot{a} > 0$$

For inflation require material with negative pressure. Not many examples. One is a scalar field!

Intro fundamental scalar field -- like Higgs

If Universe is dominated by the potential of the field, it will accelerate!



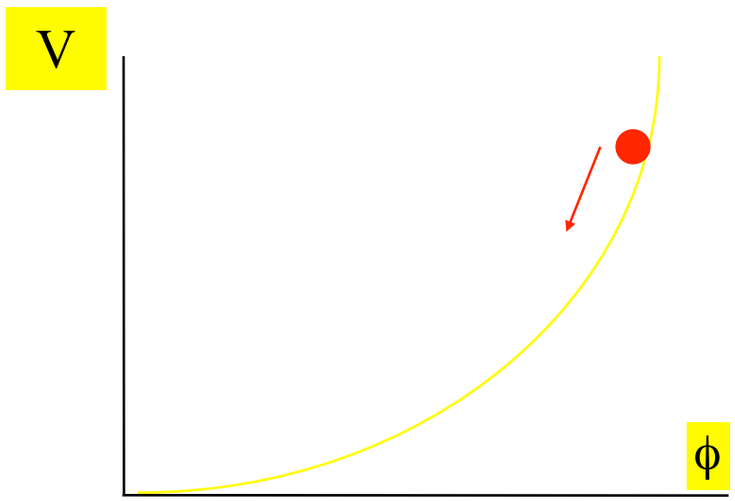

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$
$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Of course no fundamental scalar field ever seen.

We aim to constrain potential from observations.

During inflation as field slowly rolls down its potential, it undergoes quantum fluctuations which are imprinted in the Universe. Also leads to gravitational wave production.

# Examples of inflation




Simplest case – single scalar field

$$\rho_\phi = V(\phi) + \frac{\dot{\phi}^2}{2} \quad ; \quad p_\phi = \frac{\dot{\phi}^2}{2} - V(\phi)$$

EoM $$H^2 = \frac{8\pi G}{3}\rho_\phi \quad ; \quad \ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Inflation $$\ddot{a} > 0 \leftrightarrow (\rho + 3p) < 0 \leftrightarrow \dot{\phi}^2 \ll V(\phi)$$ Slow roll approx

$$\rightarrow \quad H^2 = \frac{8\pi G}{3}V(\phi) \quad ; \quad 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Also: $$\dot{H} = -4\pi G\dot{\phi}^2,$$

So, define a quantity which specifies how fast H changes during inflation

Prediction -- potential determines important quantities

Slow roll parameters [Liddle & Lyth 1992]

$$\epsilon = \frac{1}{16\pi G}\left[\frac{V'(\phi)}{V(\phi)}\right]^2$$

$$\eta = \frac{1}{8\pi G}\left[\frac{V''(\phi)}{V(\phi)}\right]$$

Inflation occurs when both of these slow roll conditions are << 1

End of inflation corresponds to ε=1
How much does the universe expand? Given by number of e-folds

$$N \equiv \ln\left(\frac{a(t_{\rm end})}{a(t_i)}\right) = \int_{t_i}^{t_e} H\,dt \simeq \int_{\phi_i}^{\phi_e} \frac{V}{V'}\,d\phi$$

Last expression is true in the slow roll limit (for single field inflation).

# Number of e-folds required

Solve say the Flatness problem:

Assume inflation until tend = $10^{-34}$ sec

Assume immediate radn dom until today, $t_0 = 10^{17}$ sec

Assume $$\left|\Omega\ (t_0) - 1\right| \leq 0.01$$

Now $$\left|\Omega - 1\right| = \frac{k}{a^2 H^2};\quad RD: \left|\Omega - 1\right| \propto t$$

$$\left|\Omega\,(10^{-34}\ \mathrm{s}) - 1\right| \leq 0.01 * 10^{-34} * 10^{-17} \leq 10^{-54}$$

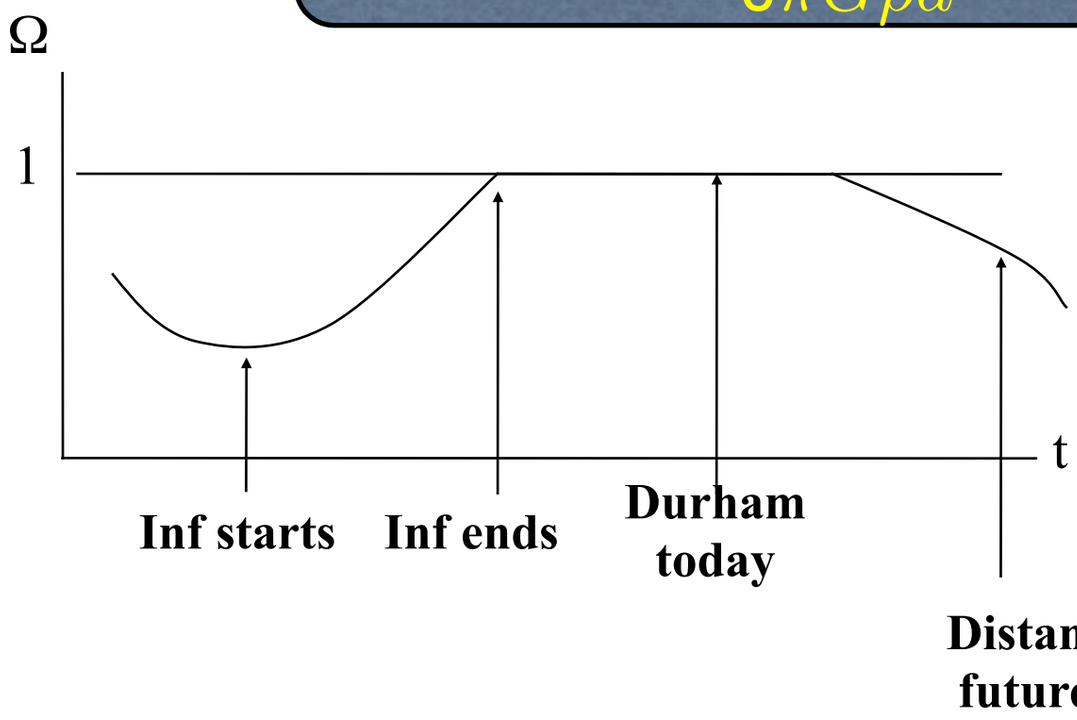
Inf $$\left|\Omega - 1\right| \propto 1/a^2 \rightarrow \frac{\left|\Omega_{end} - 1\right|}{\left|\Omega_{ini} - 1\right|} = a_i^2/a_e^2 = 10^{-54}$$

$$\rightarrow \quad N = \ln\left[\frac{a_{tend}}{a_{tini}}\right] \approx 62$$

# Solving the big bang problems

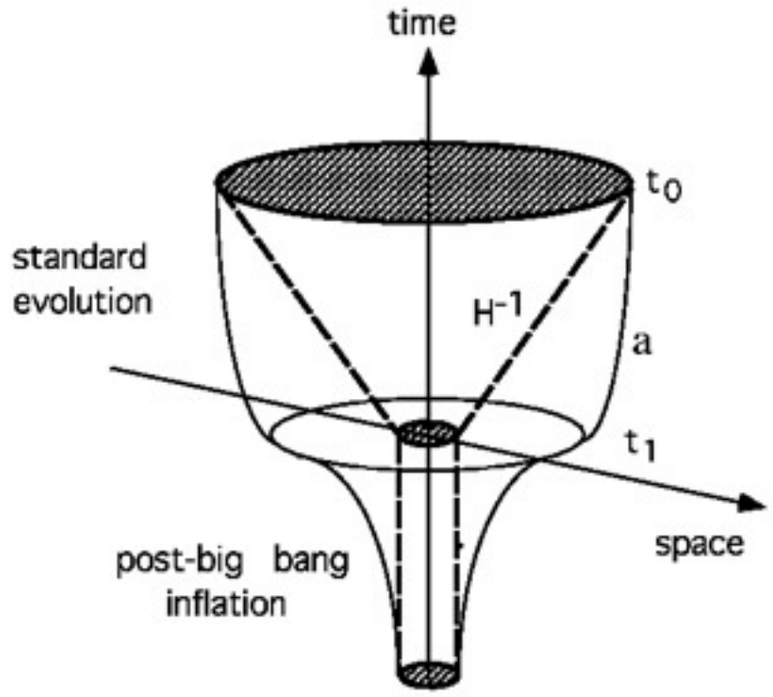
1. Flatness

$$\Omega^{-1} - 1 = -\frac{3k}{8\pi G\rho a^2} \propto a^{-2} \longrightarrow \exp(-2Ht)$$



2. Horizon problem:

Physical: $H^{-1}$ const during inflation.




Initial causally connected region

3. Monopole problem: $\rho_{\rm mon} \propto a^{-3} \longrightarrow 0$ rapidly during inflation

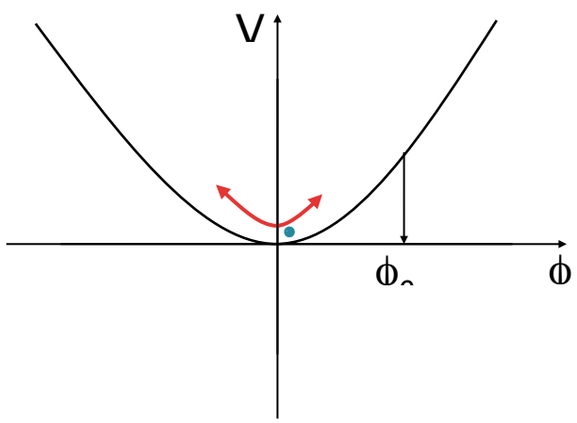
Everything infact diluted away except for the inflaton field itself.

$T \propto a^{-1} \longrightarrow 0$ Hence need to reheat the universe at end of inflation

# End of inflation

- Eventually SRA breaks down, as inflaton rolls to minima of its potential.

Experimental test of slow roll approximation – Aspen 2002




- Leaves a cold empty Universe apart from inflaton.
- Inflation has to end and the energy density of the inflaton field decays into particles. This is reheating and happens as the field oscillates around the minimum of the potential

# End of inflation.

- Inflaton is coupled to other matter fields and as it rolls down to the minima it produces particles –perturbatively or through parametric resonance where the field produces many particles in a few oscillations.

- Dramatic consequences. Universe reheats, can restore previously broken symmetries, create defects again, lead to Higgs windings and sphaleron effects, generation of baryon asymmetry at ewk scale at end of a period of inflation.

- Important constraints: e.g.: gravitino production means : $T_{rh} < 10^9$ GeV -- often a problem!

## Perturbative Reheating:

1. Instantaneous reheating where vac energy is converted immediately to radiation with $T_{RH.}$

2. Reheat by slow decay of $\phi$ with the zero modes comoving energy density decaying into particles which scatter and thermalise. Assume decay width for this is same as for free $\phi$.

Expect small decay width, as flatness of potential requires weak coupling of $\phi$ to other fields. Also in SUGR if coupling not weak, overproduce gravitinos during reheating.

Boltzmann eqn:

$$\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0$$
$$\dot{\rho}_{\rm rad} + 4H\rho_{\rm rad} - \Gamma_\phi \rho_\phi = 0$$

$T_{RH}$ – inflaton executes coherent oscillations about $V_{min}$ after inflation.

$< \rho_\phi >_{osc} \propto a^{-3}$ Averaged over many coherent oscillations

$\rho_{\phi I}, a_I$ Values when coherent oscillations start.

Hubble expansion rate: $$H(a) = \sqrt{\frac{8\pi G}{3}\rho_{\phi I}\left(\frac{a_I}{a}\right)^3}$$

Equating: $H(a) = \Gamma_\phi$ gives $$\left(\frac{a_I}{a}\right) = \left(\frac{3G\Gamma_\phi^2}{8\pi\rho_{\phi I}}\right)^{1/3}$$

Assume at this moment all coherent energy density immediately transferred into radiation.

$$\rho_\phi = \rho_R \text{ where } \rho_\phi = \rho_{\phi I}\left(\frac{a_I}{a}\right)^3 \text{ and } \rho_R = \left(\frac{\pi^2}{30}\right)g_* T_{RH}^4$$

Hence: $$T_{RH} = \left(\frac{90}{8\pi^3 g_*}\right)^{1/4}\sqrt{\Gamma_\phi M_P} = 0.2\left(\frac{200}{g_*}\right)^{1/4}\sqrt{\Gamma_\phi M_P}$$

Bound from Gravitino overproduction : $T_{RH} \le 10^9 - 10^{10}\,\text{GeV}$

**Preheating:** **Traschen & Brandenberger; Kofman, Linde & Starobinsky**

Non-perturbative resonant transfer of energy to particles induced by the coherent oscillations of $\phi$ -- can be very efficient!

Assume $\phi$ oscillating about min of potential.

$$V(\phi) = \frac{m^2\phi^2}{2}; \text{ Write } \phi(t) = \Phi(t)\sin mt$$

In expanding universe $\Phi$ decreases due to redshift of momentum.

Assume scalar field X coupled to $\phi$ $\quad L_{int} = \frac{g^2X^2\phi^2}{2}$

Mode eqn: $\chi_k = X_k a^{3/2}$: $\quad \ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + g^2\Phi^2(t)\sin^2(mt)\right)\chi_k = 0$

Minkowski space:
$\Phi$ const

$$\chi_k'' + [A_k - 2q\cos(2z)]\chi_k = 0;$$

$$z = mt,\ \chi_k' \equiv d\chi_k/dz;\ q = \frac{g^2\Phi^2}{4m^2};\ A_k = 2q + \frac{k^2}{m^2}$$

Mathieu equation

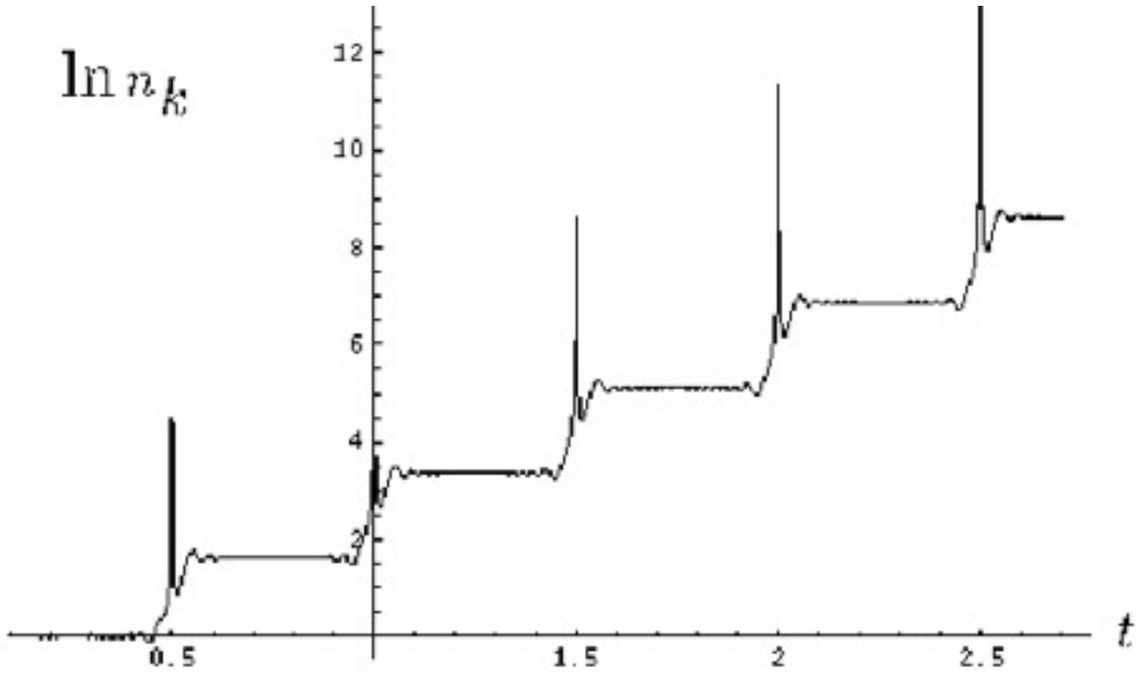
Exponential instability regions:

$$\chi_k \propto \exp(\mu_k z) \text{ where } \mu_k = \sqrt{\left(\frac{q}{2}\right)^2 - \left(\frac{2k}{m} - 1\right)^2}$$

Max growth at $2k = m$

Growth of modes leads to growth of occupation numbers of created particles

Number density = Energy of that mode/Energy of each particle ($\omega_k$)

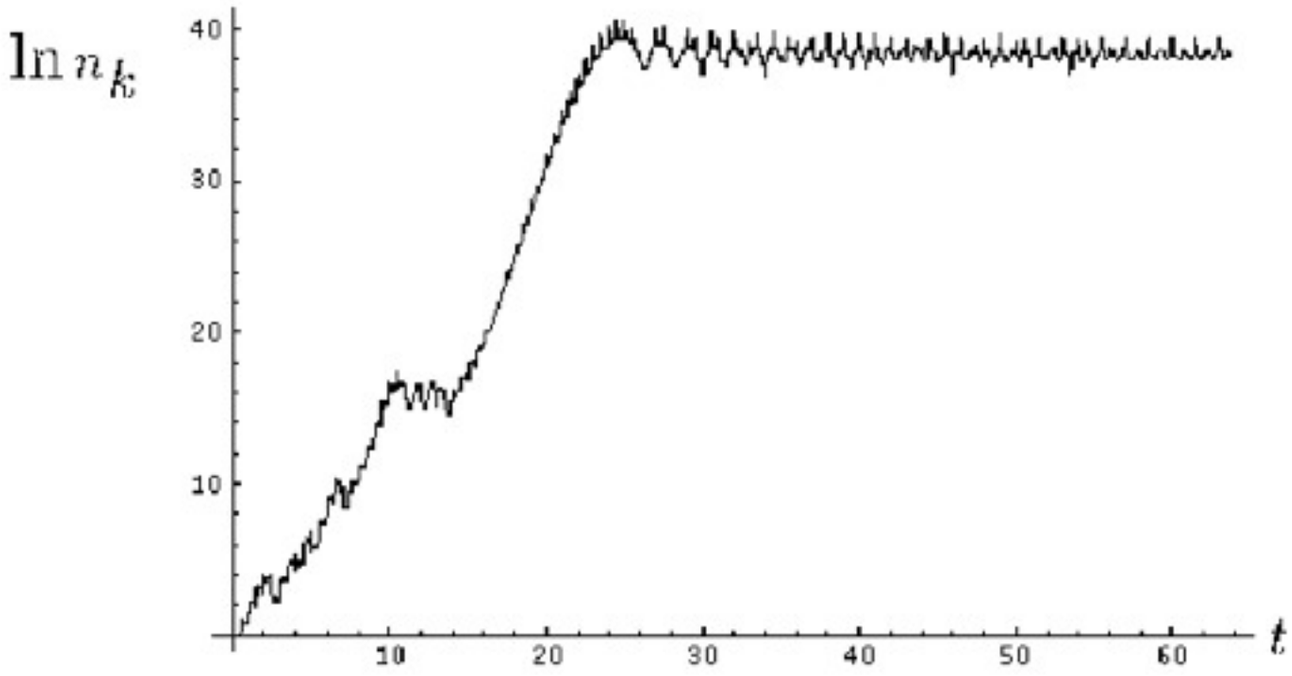



Kofman, Linde and Starobinsky (97)

Period of enhanced rate of energy transfer – preheating, because particles produced not in thermal eqm. Explosive growth every time $\phi(t)=0$.

$$n_k \propto \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2} \propto e^{2\mu_k z}$$

Still occurs when A,q not constant:




**Kofman, Linde and Starobinsky (97)**

Longer time evolution

This efficient quick transfer of energy means that can have large reheat temperatures, phase transitions, defect production and baryogenesis through production of particles with mass bigger than inflaton mass. Can also generate potentially obervable primordial gravitational waves from pre-heating.

# The origins of perturbations -- the most important aspect of inflation

Idea: Inflaton field is subject to perturbations (quantum and thermal fluctuations). Those are stretched to superhorizon scales, where they become classical. They induce metric perturbations which in turn become later the first perturbations to seed the structures in the universe.

Also predict a cosmological gravitational wave background.

During inf $\phi(\underline{x}, t) = \phi_0(t) + \delta\phi(\underline{x}, t)$ ← Quantum fluc

Fourier modes: $\delta\phi(\underline{x}, t) = \sum_k \delta\phi_k(t)\, e^{i k x}$ → Generates fluc in matter and metric

$\delta_H^2(k)$ Scalar pertn – spectra of gaussian adiabatic density pertns generated by flucns in scalar field and spacetime metric. Responsible for structure formation.

$h_{\mu\nu}$ $A_G(k)$ Tensor pertn in metric– gravitational waves.

# Key features

During inflation comoving Hubble length (1/aH) decreases.

So, a given comoving scale can start inside (1/aH), be affected by causal physics, then later leave (1/aH) with the pertns generated being imprinted.

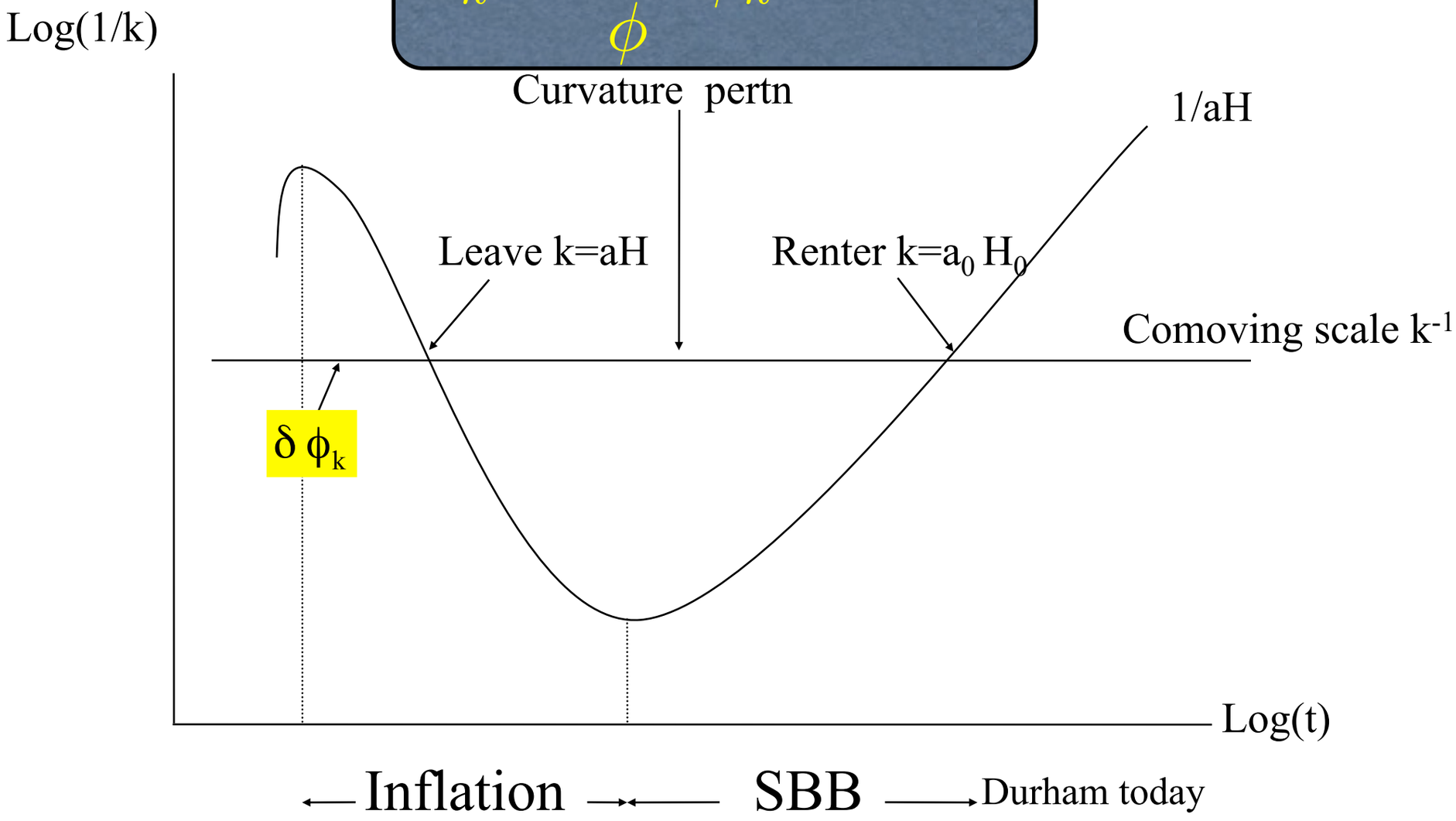
Quantum flucns in inflaton arise from uncertainty principle.

Pertns are created on wide range of scales and generated causally.

Size of irregularities depend on energy scale at which inflation occurs.

Pertn created causally, stretched by expansion.

$$\mathcal{R}_k = \frac{H}{\dot{\phi}} \delta\phi_k \simeq \text{const}$$



# The power spectra

Focus on statistical measures of clustering.

Inflation predicts amp of waves of a given k obey gaussian statistics, the amplitude of each wave chosen independently and randomly from its gaussian. It predicts how the amplitude varies with scale — the power spectrum

Good approx -- power spectra as being power-laws with scale.

Density pertn

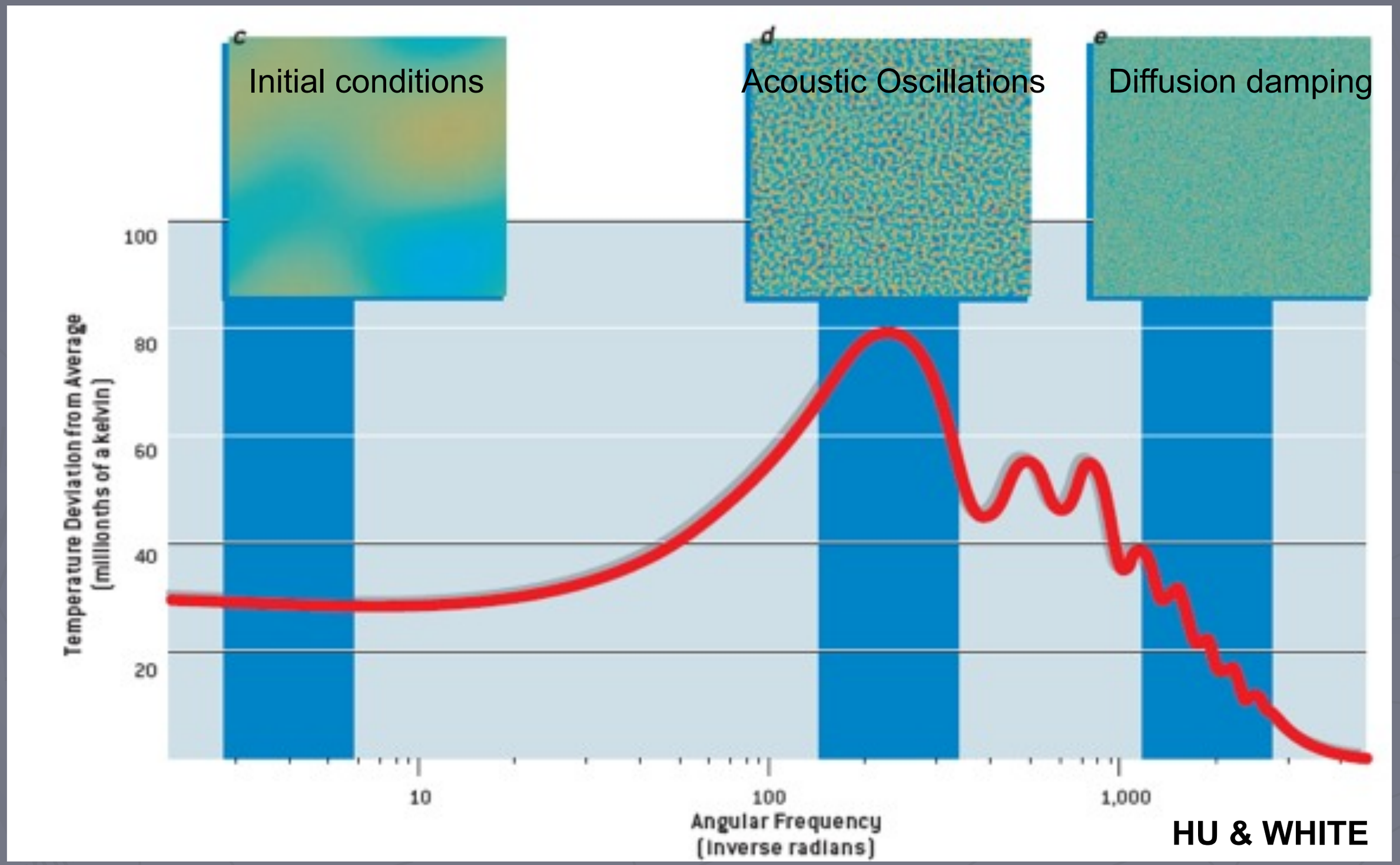
$$\delta_H^2(k) = \delta_H^2(k_0)\left[\frac{k}{k_0}\right]^{n-1}$$

Grav waves

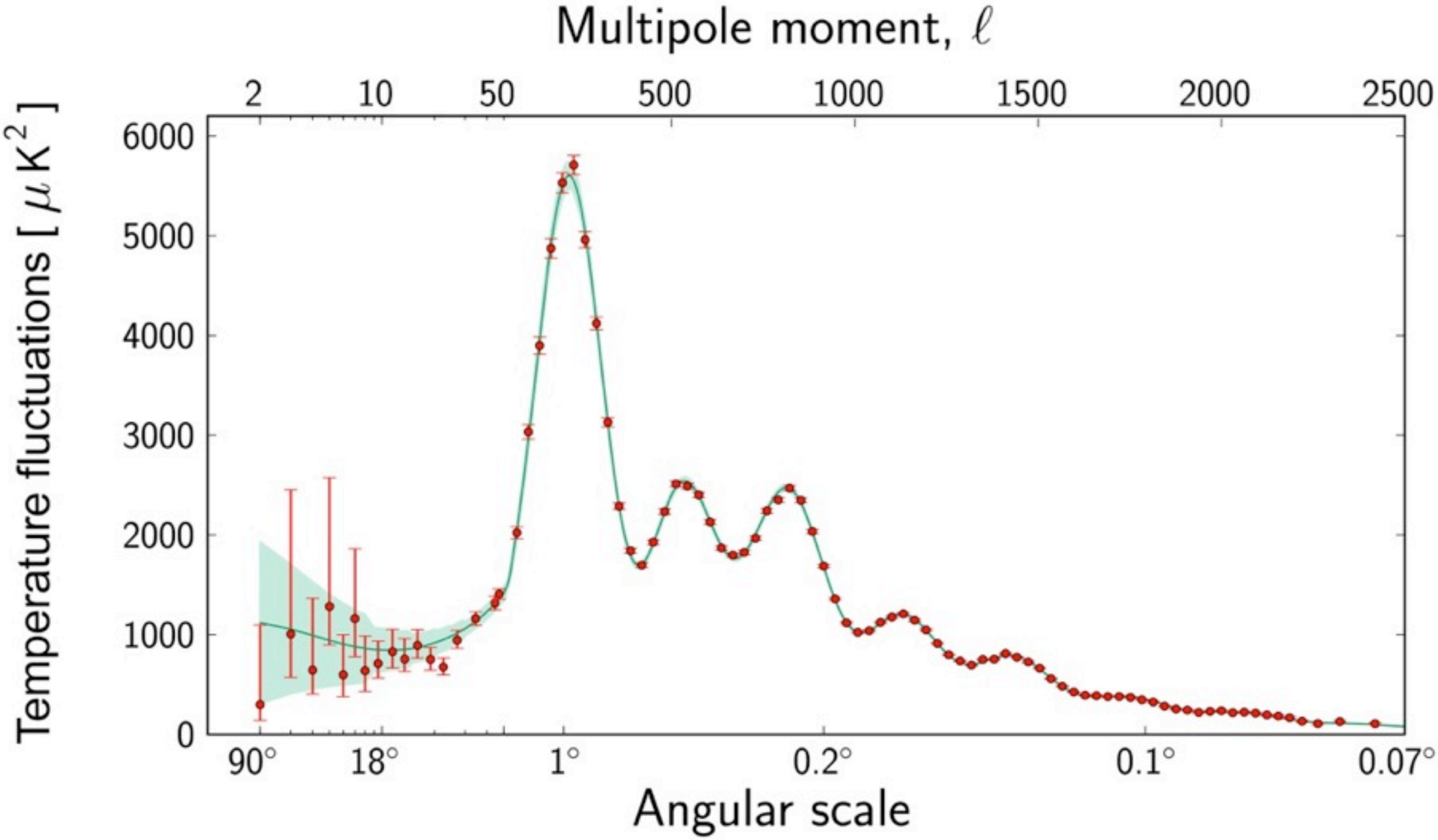
$$A_G^2(k) = A_G^2(k_0)\left[\frac{k}{k_0}\right]^{n_G}$$

Four parameters

# Temperature Power spectrum



# Power spectrum - LCDM fit



# Some formulae

Power spectra $$P_\phi(k) = \frac{k^3}{2\pi^2}\left\langle \left|\delta\phi_k\right|^2 \right\rangle$$

Vacuum soln $$\left\langle \left|\delta\phi_k\right|^2 \right\rangle = \frac{H^2}{2k^3} \rightarrow P_\phi(k) = \left|\frac{H}{2\pi}\right|^2_{k=aH(Exit)}$$

Amp of density pertn $$\delta_H^2(k) = \frac{4}{25}\left(\frac{H}{\dot{\phi}}\right)^2\left(\frac{H}{2\pi}\right)^2_{k=aH}$$

SRA $$\delta_H(k) \propto \kappa^{3/2}\frac{V^{3/2}}{|V'|}\Big]_{k=aH}$$

WMAP: 60 efolds before tend $\rightarrow$ $\delta_H(k) \approx 1.91 * 10^{-5}$

$\rightarrow$ $V^{1/4}/\varepsilon \leq 10^{16}$ GeV – – Lyth

In other words the properties of the inflationary potential are constrained by the CMB

Tensor pertns : amp of grav waves.

$$A_G(k) \propto \kappa^2 V^{1/2} \Big]_{k=aH}$$

Note: Amp of perts depends on form of potential.
Tensor pertns gives info directly on potential but difficult to detect.

# Observational consequences.

Precision CMBR expts like WMAP and Planck → probing spectra.

Standard approx – power law.

$$\delta_H^2(k) \propto k^{n-1} \; ; \; A_G^2(k) \propto k^{n_G}$$

$$n - 1 = \frac{d \ln \delta_H^2}{d \ln k} \; ; \; n_G = \frac{d \ln A_G^2}{d \ln k}$$

Power law ok, only a limited range of scales are observable.

For range 1Mpc → $10^4$ Mpc : $\Delta \ln k \approx 9$

Crucial eqn

$$\frac{d \ln k}{d \phi} = \kappa \frac{V}{V'} \quad \Rightarrow \quad n = 1 - 6\varepsilon + 2\eta \; ; \; n_G = -2\varepsilon$$

n=1 ; $n_G$=0 – Harrison Zeldovich

CMBR → Measure relative importance of density pertns and grav waves.

$$R = \frac{C_2^{GW}}{C_2^{S}} \approx 4\pi\varepsilon$$

$$\text{where } \frac{\Delta T}{T} = \sum a_{lm} Y_m^l(\theta,\varphi),\ C_l = \left\langle |a_{lm}|^2 \right\rangle$$

$C_l$ -- radiation angular power spectrum.

A unique test of inflation $R = -2\,\pi\, n_G$

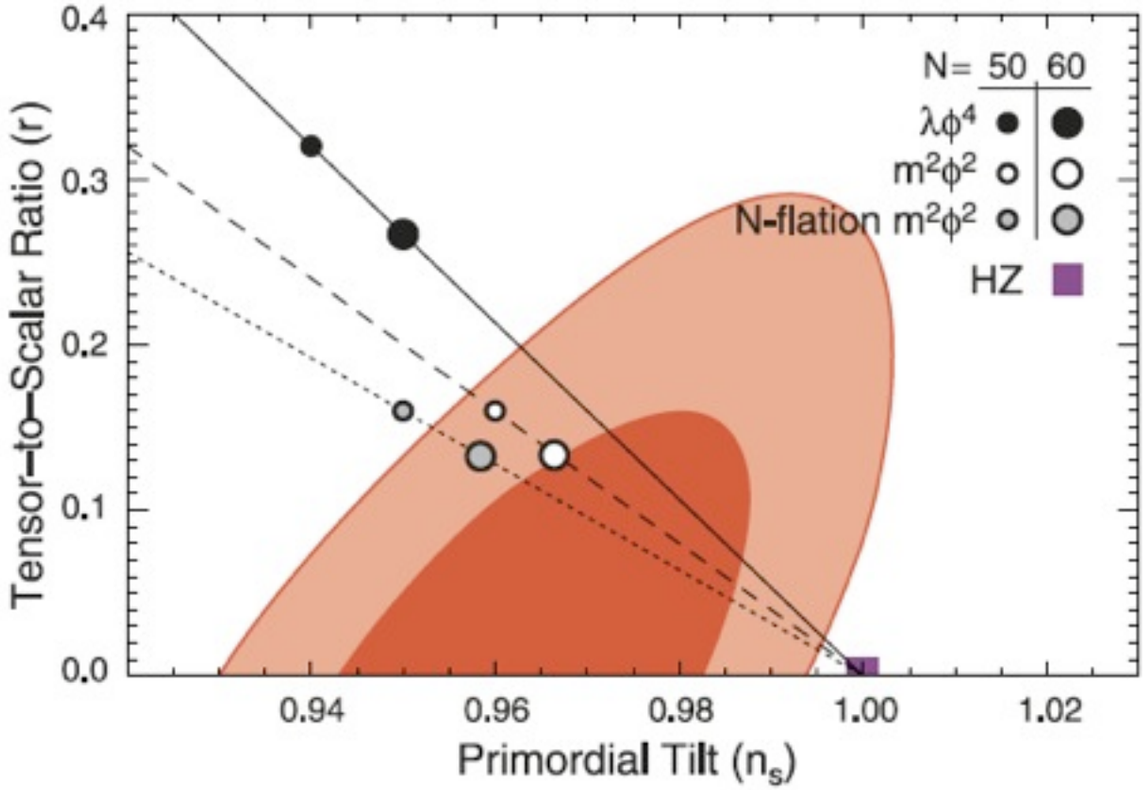
Indep of choice of inf model, relies on slow roll and power law approx. Unfortunately $n_G$ too small for detection, but maybe Planck !

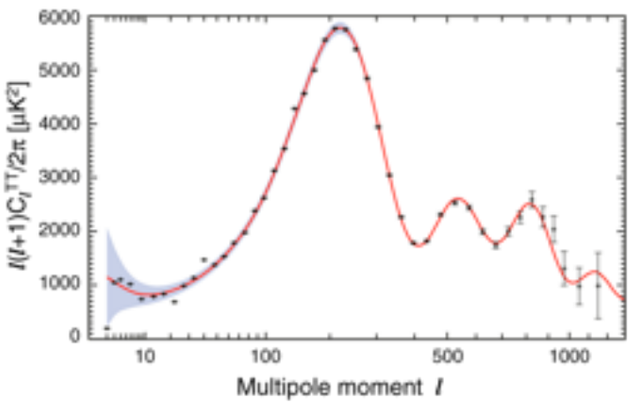
Example if include WMAP7+BAO+H0 constraints:

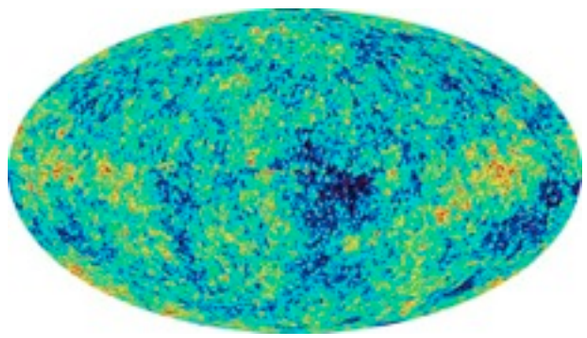
$k_0 = 0.002\mathrm{Mpc}^{-1}$

No GW assumed: $n_s = 0.963 \pm 0.012$

Allow for GW: $n_s = 0.973 \pm 0.014$, $r < 0.24\ (95\%\ CL)$



**(Komatsu et al, 2010)**

# Some examples – Chaotic Inflation

$$V(\phi) = \frac{m^2\phi^2}{2} \qquad \text{with} \qquad \varepsilon = \frac{1}{2\kappa^2}\left[\frac{V'}{V}\right]^2 \;\; ; \;\; \eta = \frac{1}{\kappa^2}\left[\frac{V''}{V}\right]$$

Find: $$\varepsilon = \frac{2}{\kappa^2\phi^2} = \eta$$

SRA: $$H^2 = \frac{8\pi G}{3}V(\phi) \quad ; \quad 3H\dot{\phi} + \frac{dV}{d\phi} = 0$$

Inf soln:

$$\phi(t) = \phi_i - \frac{\sqrt{2}mt}{\sqrt{3}\kappa} \quad ;$$

$$a(t) = a_i \exp\left[\frac{\kappa m}{\sqrt{6}}\left(\phi_i t - \frac{mt^2}{\sqrt{6}\kappa}\right)\right]$$

End of inflation: $$\varepsilon = 1 \Rightarrow \phi_e = \frac{\sqrt{2}}{\kappa}$$

Num of e-folds: $$N(\phi) = -\kappa^2 \int_{\phi}^{\phi_e} \frac{V}{V'}\, d\phi = \frac{\kappa^2 \phi^2}{4} - \frac{1}{2}$$

N=60: $$\phi_{60} \approx \frac{16}{\kappa} > \phi_e$$

Scale just entering Hubble radius today, COBE scale

Amp of den pertn: $$\delta_H(k) = \frac{\kappa^3}{\sqrt{75\pi}} \frac{V^{3/2}}{|V'|}\Bigg]_{k=aH}$$

Take to be 60 efolds before end of inflation.

Find: $$\delta_H(k) = 12m\sqrt{G} \quad \text{where} \quad \kappa^2 \equiv 8\pi G$$

Amp of grav waves: $$A_G(k) = \sqrt{\frac{32}{75}}\, G V^{1/2} \Big]_{k=aH}$$ 60 efolds before end of inflation.

Find: $$A_G(k) \approx 1.4 m \sqrt{G}$$

Normalise to COBE: $$\delta_H(k) \approx 1.91 * 10^{-5}$$

Find: $$m = 2 * 10^{13}\ \text{GeV}$$ **Constraint on inflaton mass!**

Spectral indices $$n = 1 - 6\varepsilon + 2\eta\,;\, n_G = -2\varepsilon$$ Slow roll

Use values 60 e-folds before end of inflation.

$$n = 0.97;\, n_G = -0.016$$ Close to scale inv

# Digression Key features in Planck

- Its a bit strange -- the standard ΛCDM model subject to almost scale free Gaussian fluctuations in the early universe appears to work really well especially on small scales but ...
- There are some anomalies which although maybe at low signifcance could well be hints that all is not well, there is new physics lurking in there.
- Clear evidence of tilt in spectra: $n_s$ now 6σ away from $n_s$=1
- Potentially important difference with previous analysis: lower $H_0$, higher $\Omega_{m,}$ the universe is a bit older than we thought.
- No evidence for primordial NG
- Hints of features on small angular scales, temp differences in N and S hemispheres.
- Fewer clusters observed (factor of 2) in CMB compared to what predicted by ΛCDM

# Tensions: Planck v WMAP + SPT

| Parameter | *Planck*+WP+highL | | *WMAP*7+SPT (S12) | |
|---|---|---|---|---|
| | Best fit | 68% limit | Best fit | 68% limits |
| $100\Omega_b h^2$ . . . . . | 2.207 | $2.207 \pm 0.027$ | 2.223 | $2.229 \pm 0.037$ |
| $\Omega_c h^2$ . . . . . | 0.1203 | $0.1198 \pm 0.0026$ | 0.1097 | $0.1093 \pm 0.0040$ |
| $10^9 A_s$ . . . . . | 2.211 | $2.198 \pm 0.056$ | 2.143 | $2.142 \pm 0.061$ |
| $n_s$ . . . . . | 0.958 | $0.959 \pm 0.007$ | 0.963 | $0.962 \pm 0.010$ |
| $\tau$ . . . . . | 0.093 | $0.091 \pm 0.014$ | 0.083 | $0.083 \pm 0.014$ |
| $100\theta_*$ . . . . . | 1.0414 | $1.0415 \pm 0.0006$ | 1.0425 | $1.0429 \pm 0.0010$ |
| $\Omega_\Lambda$ . . . . . | 0.683 | $0.685 \pm 0.017$ | 0.747 | $0.750 \pm 0.020$ |
| $H_0$ . . . . . | 67.2 | $67.3 \pm 1.2$ | 72.3 | $72.5 \pm 1.9$ |

- Problem traced to calibration of SPT in overlap region with WMAP $600<l<1000$ where data noisy.
- For high l use Planck + ACT data and SPT for $l>2000$

# Hubble constant

- Hubble constant low : $H_0 = 67.3 \pm 1.2\ \mathrm{kms^{-1}Mpc^{-1}}$
- Model dependent - degeneracy with dark energy eos.
- Local measurements: HST (Riess et al 2011) $H_0 = 73.8 \pm 2.4\ \mathrm{kms^{-1}Mpc^{-1}}$
- Carnegie Hubble programme (Freedman et al 2012)

$$H_0 = 74.3 \pm 1.5\ (\text{statistical}) \pm 2.1\ (\text{systematic})\ \mathrm{kms^{-1}Mpc^{-1}}$$

- Local measurements in 2.5σ tension with Planck
- New physics, problems with local measurements?
- Maybe global and local values for $H_0$ really are different ! Would see this effect if we lived in a slightly underdense part of the universe, matter would flow outwards and we would interpret as higher $H_0$
- Note, low value completely consistent with Baryon Acoustic Oscillation values -- geometric probe of distance scales, means to break degenaracies.
- Need to be careful when combining Planck + SN data !

# Scale dependence of fluctuations

- For scales 2<l<2000, 6σ deviation from scale invariance.

$$n_{\rm s} = 0.958 \pm 0.007 \quad (68\%, \textit{Planck}{+}\text{WP}{+}\text{highL})$$

- Power Spectrum parameterised by:

$$\mathcal{P}_{\mathcal{R}}(k) = A_{\rm s}\left(\frac{k}{k_0}\right)^{n_{\rm s}-1+(1/2)(dn_{\rm s}/d\ln k)\ln(k/k_0)}$$

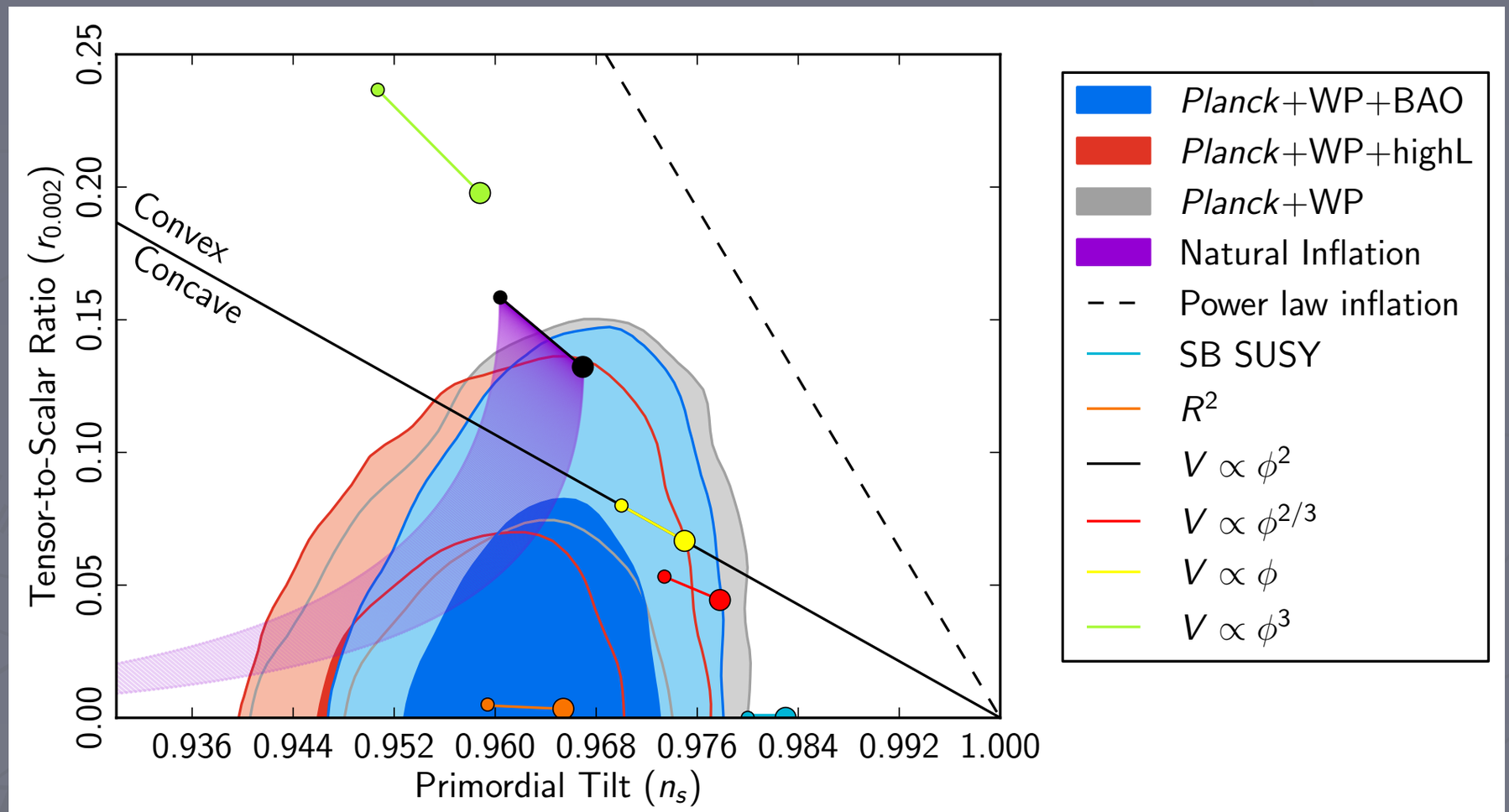
- Find small negative running but low significance

$$dn_{\rm s}/d\ln k = -0.015 \pm 0.009 \ (68\%, \textit{Planck}{+}\text{WP}{+}\text{highL}),$$

- Planck breaks degeneracy between spectral index and tensor to scalar ratio
- r< 0.11
- Scale of inflation $V_* = \dfrac{3A_S\, r\, m_{\rm P}^4}{128} < 6.4\times10^{-12} m_{\rm P}^4 = (1.94\times10^{16}{\rm G}eV)^4$
- Something interesting happening at high l~1700 - drop in power, but not clear it is significant yet.

# Slow roll results

- Reheating/flatness constrains $50 < N_* < 60$ Planck + WP




- Power law - (n=3,4 ruled out). Other interesting models include n=1, n=2/3 (axial monodromy)
- Power law (exponential potential) ruled out

# Neutrino species

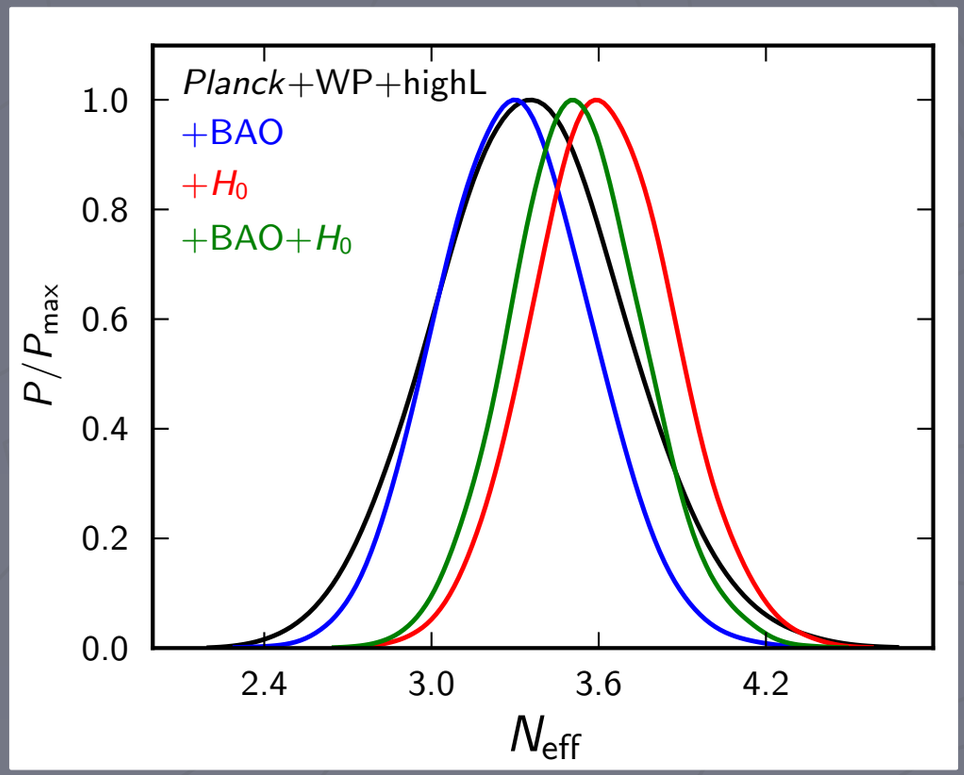
- Account for energy density of neutrinos by

$$\rho_\nu = N_{\rm eff}\,\frac{7}{8}\left(\frac{4}{11}\right)^{4/3}\rho_\gamma$$

- Standard model predicts $N_{eff} = 3.046$
- Main affect on CMB is increasing expansion rate before recombination - reduced power in damping tail
- WMAP + SPT suggested extra relativistic species at ~2σ level
- Planck and BAO consistent with standard value at 1σ level - no evidence for extra relativistic degrees of freedom
- $N_{eff}$ is correlated with $H_0$ - tension with of LCDM with local $H_0$ can be relieved with higher $N_{eff}$

$$N_{\rm eff} = 3.62^{+0.50}_{-0.48} \quad (95\%;\ \textit{Planck}{+}\mathrm{WP}{+}\mathrm{highL}{+}H_0).$$

- No preference for this model from CMB



# Dark Energy

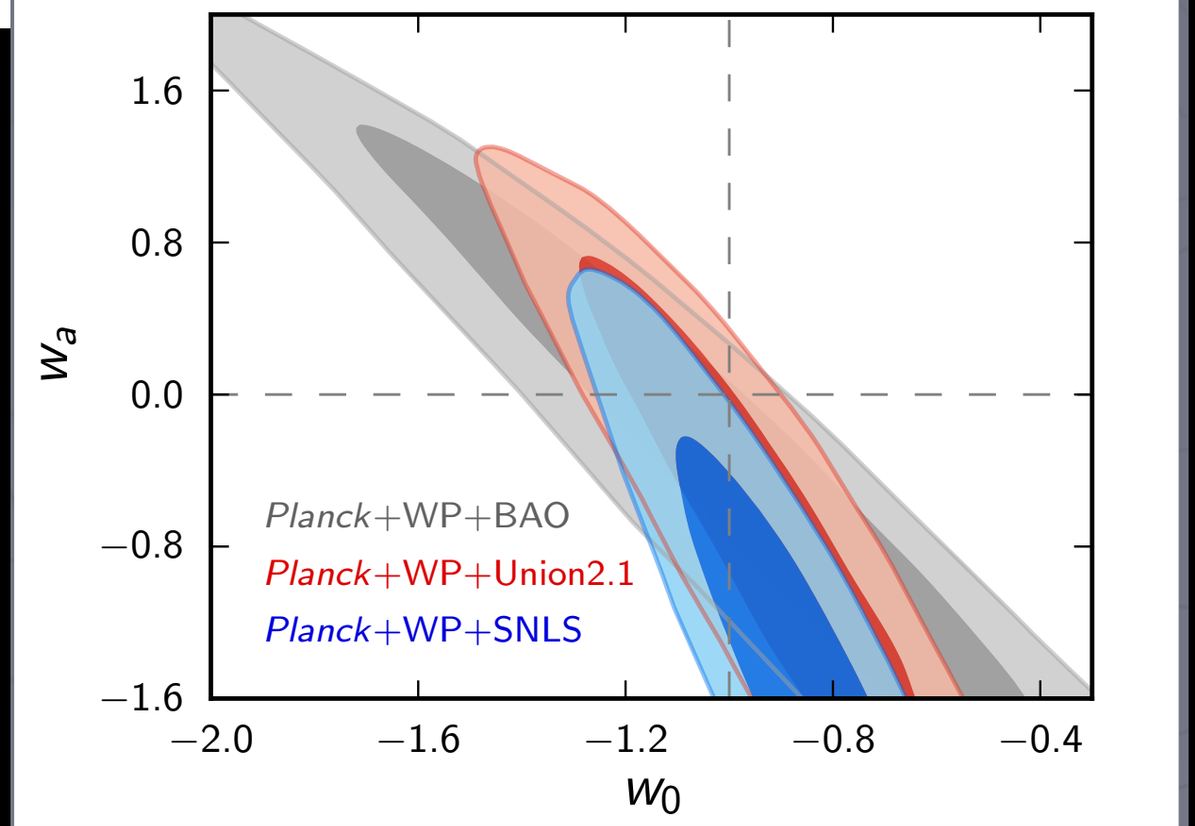
Parameterise eos: $$w(a) \equiv \frac{p}{\rho} = w_0 + (1-a)w_a$$

Planck alone weak constraints on DE because of degeneracy of w with $H_0$:

Break with other probes including lensing, SN, BAO ...

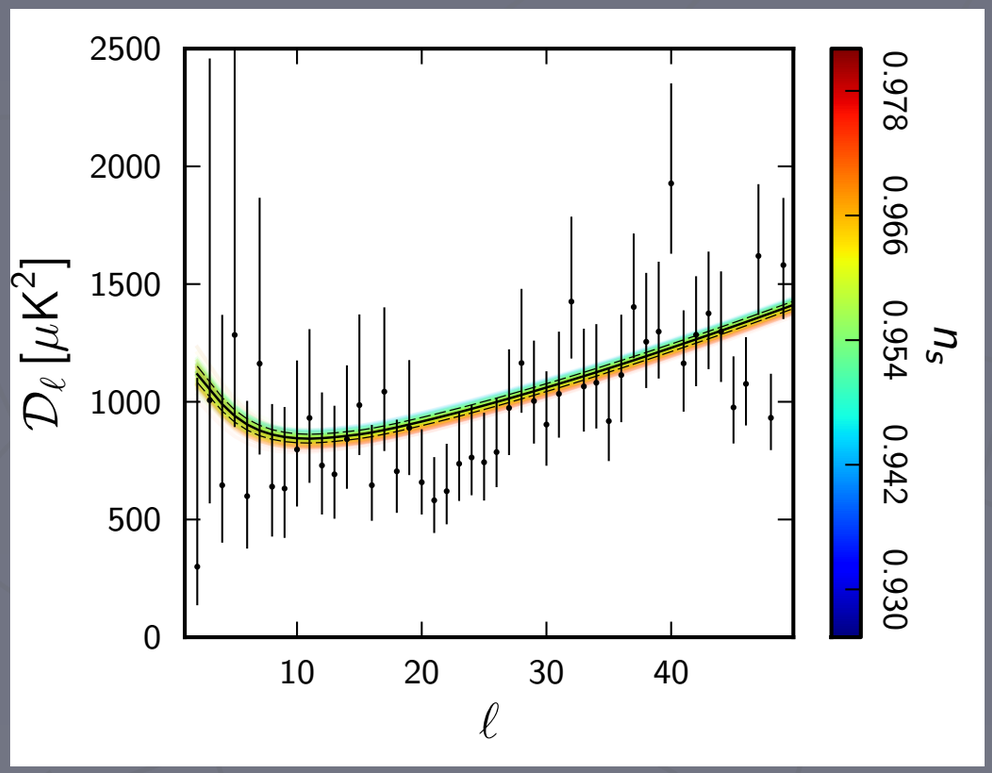
Example - if assume $w_a = 0$

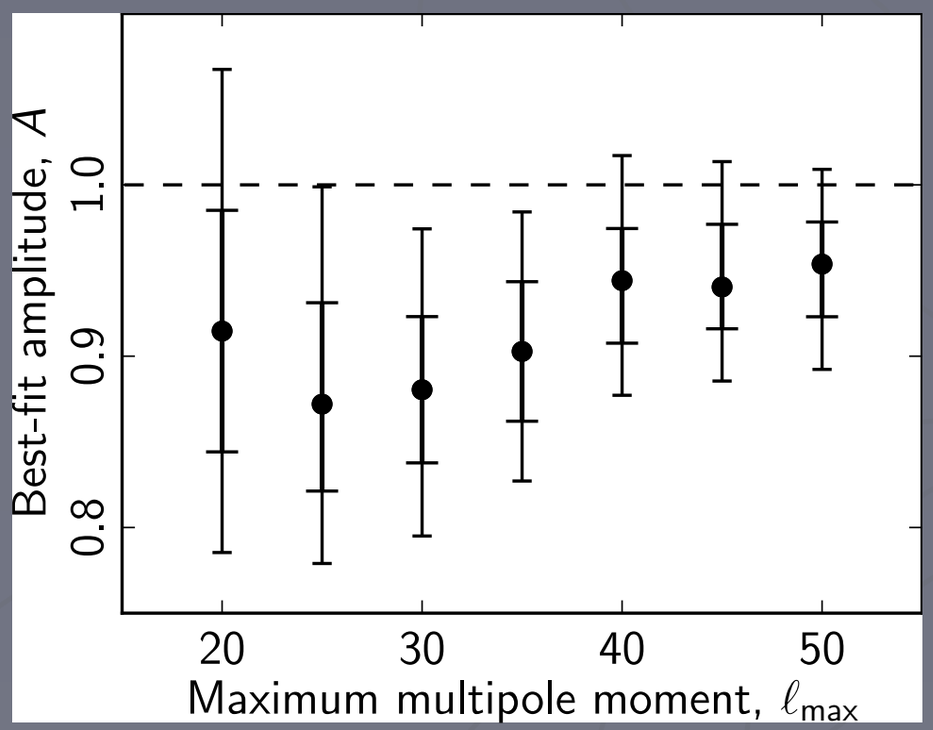
$$w = -1.13 \pm 0.24 \quad (95\%, \textit{Planck} + \text{WP} + \text{BAO})$$
$$w = -1.09 \pm 0.17 \quad (95\%, \textit{Planck} + \text{WP} + \text{Union2.1})$$
$$w = -1.13^{+0.13}_{-0.14} \quad (95\%, \textit{Planck} + \text{WP} + \text{SNLS}),$$
$$w = -1.24^{+0.18}_{-0.19} \quad (95\%, \textit{Planck} + \text{WP} + \text{HST})$$



# Anomalies in low mulitpole spectrum

Measured power spectrum shows dip relative to best fit LCDM for 20<l<30




Moss 2013

Agrees with WMAP observation, real feature of CMB

New physics at play here? Step in inflation potential leading to feature?

# Non-Gaussianity

Single field inflation produces negligible levels of primordial NG - a detection would rule out this large class of models.

Multi-field models produce NG with different amplitudes, shapes and scale dependence.

3 point function (Bispectrum) - simplest statistic to measure NG

$$B_\Phi(k_1, k_2, k_3) = f_{\rm NL} F(k_1, k_2, k_3)$$

Different k relations lead to local, orthogonal and equilateral types of fNL:

| | **Independent** KSW | **ISW-lensing subtracted** KSW |
|---|---|---|
| SMICA | | |
| Local . . . . . | 9.8 ± 5.8 | **2.7 ± 5.8** |
| Equilateral . . . . . | −37 ± 75 | **−42 ± 75** |
| Orthogonal . . . . . | −46 ± 39 | **−25 ± 39** |

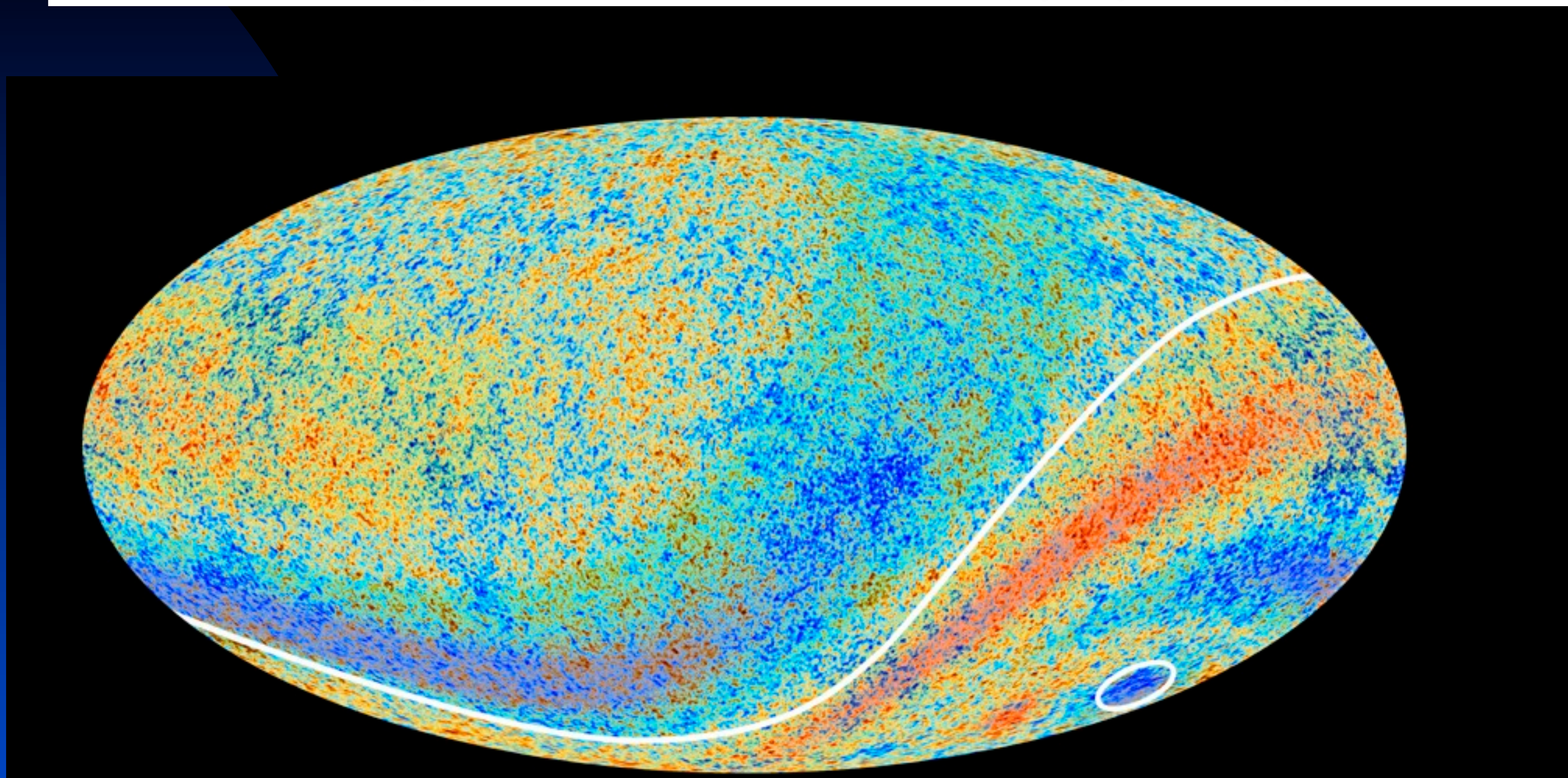
Planck finds no evidence for any of them - single slow roll inflation survives!

NG places constraints on many models, DBI inflation, curvaton all fine.

Cyclic models in their simplest form give local |fNL| > 10 so under tension with Planck.

# Dipole asymmetry

WMAP maps saw evidence for  more power in one hemisphere. Planck sees this aswell at a level of 2-3σ. Evidence of new physics? Superhorizon fluctuations during inflation, curvaton features (Lyth yesterday)?



# Cluster constraints

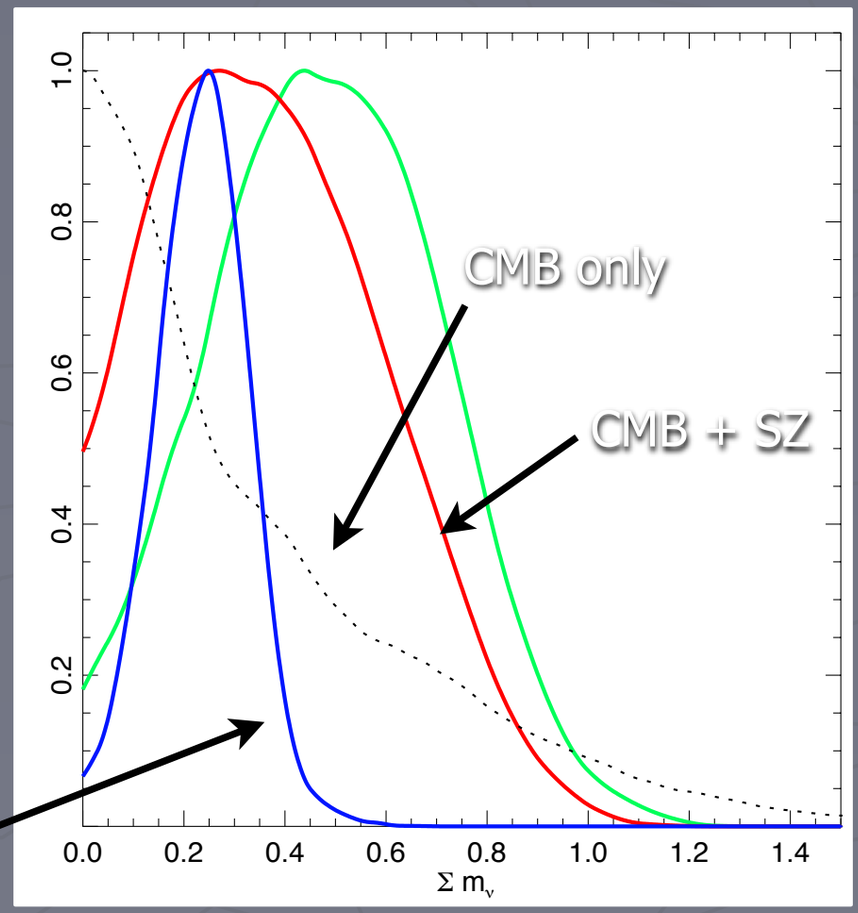
- Conclusion: LCDM model from CMB expects to see ~2 times as many clusters as observed
- Intriguing possibility: matter power spectrum is suppressed on galaxy cluster scales
- One mechanism to do this is a non-zero neutrino mass
- From CMB alone

$$\sum m_\nu < 0.66\,\mathrm{eV} \quad (95\%;\ \textit{Planck}+\mathrm{WP}+\mathrm{highL})$$

- From CMB + SZ + BAO

$$\sum m_\nu = (0.22 \pm 0.09)\,\mathrm{eV}$$

- More detailed investigation of y-parameter/ mass scaling is required, but potentially interesting cosmological result






Adam Moss 2013

# 2. Models of Inflation—variety is the spice of life.

## (where is the inflaton in particle physics?)

(Lyth and Riotto, Phys. Rep. 314, 1, (1998), Lyth and Liddle (2009)

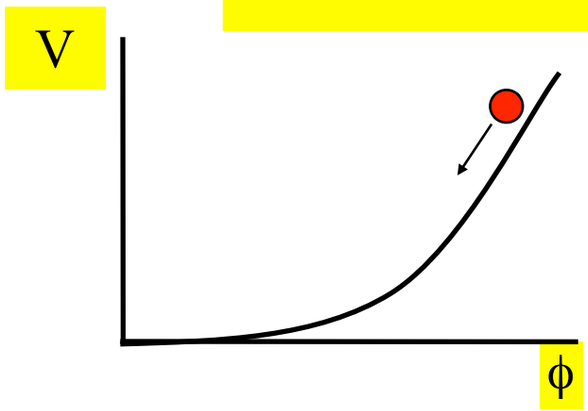
Field theory $$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2 + M\phi^3 + \lambda\phi^4 + \sum_{d=5}^{\infty}\lambda_d M_P^{4-d}\phi^d$$

Quantum corrections give coefficients proportional to $\ln(\phi)$
and an additional term proportional to $\ln(\phi)$

**1. Chaotic inflation .**

$$V(\phi) \propto \phi^p;\ \phi >> M_P;\ n-1 = -(2+p)/2N;$$

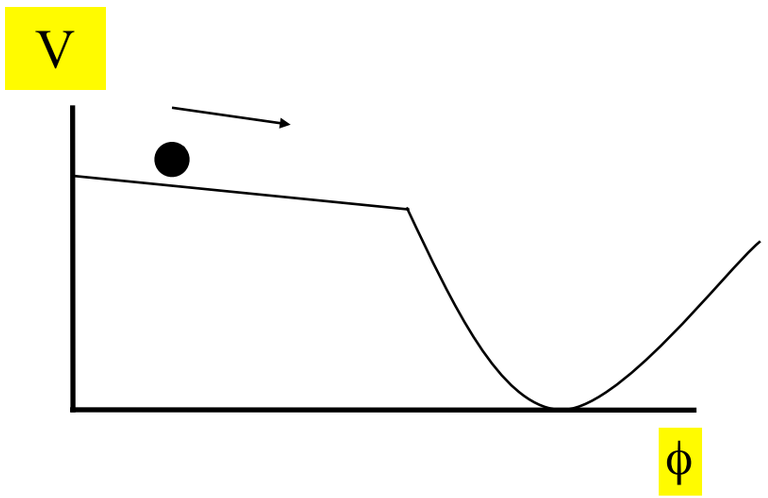
$$R = -2\pi n_G = {}^{3.1p}/_{N} \Rightarrow \text{sig grav waves.}$$




Inflates only for $\phi >> M_P$. Problem.
Why only one term? All other models inflate at $\phi < M_P$ and give negligible grav. waves.

## 2. New inflation

$$V(\phi) = V_0 - c\phi^p + \ldots;\ \ p \geq 3;\ \ n - 1 = -\frac{2(p-1)}{(p-2)N}$$




$$V(\phi) = V_0 - \frac{1}{2}m^2\phi^2 + \ldots; \Rightarrow n - 1 = -\frac{2M_P^2 m^2}{V_0}$$

$p = 2$ : modular, natural, quadratic inflation

## 3. Power-law inflation

$$V(\phi) = V_0 \exp\left(-\sqrt{\frac{16\pi}{p}}\frac{\phi}{m_P}\right);\ p > 1;\ \ n - 1 = -\frac{2}{p}$$

**1. Very useful because have exact solutions without recourse to slow roll. Similarly perturbation eqns can be solved exactly.**

**2. No natural end to inflation**

**4. Natural inflation**

$$V(\phi) = V_0\left(1 + \cos\frac{\phi}{f}\right);$$

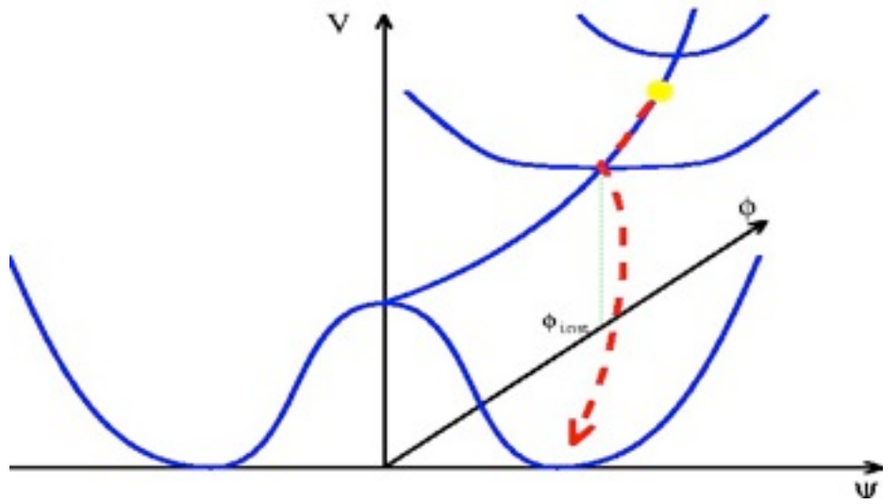
$$n - 1 < 0; \quad R - \text{negligible} - \text{-like New Inflation}$$

**5. Hybrid inflation**

$$V(\phi) = V_0 + \frac{1}{2}m^2\phi^2;$$

$$n - 1 = 2M_P^2 m^2 / V_0$$

**2 fields, inf ends when $V_0$ destabilised by 2nd non-inflaton field $\psi$**

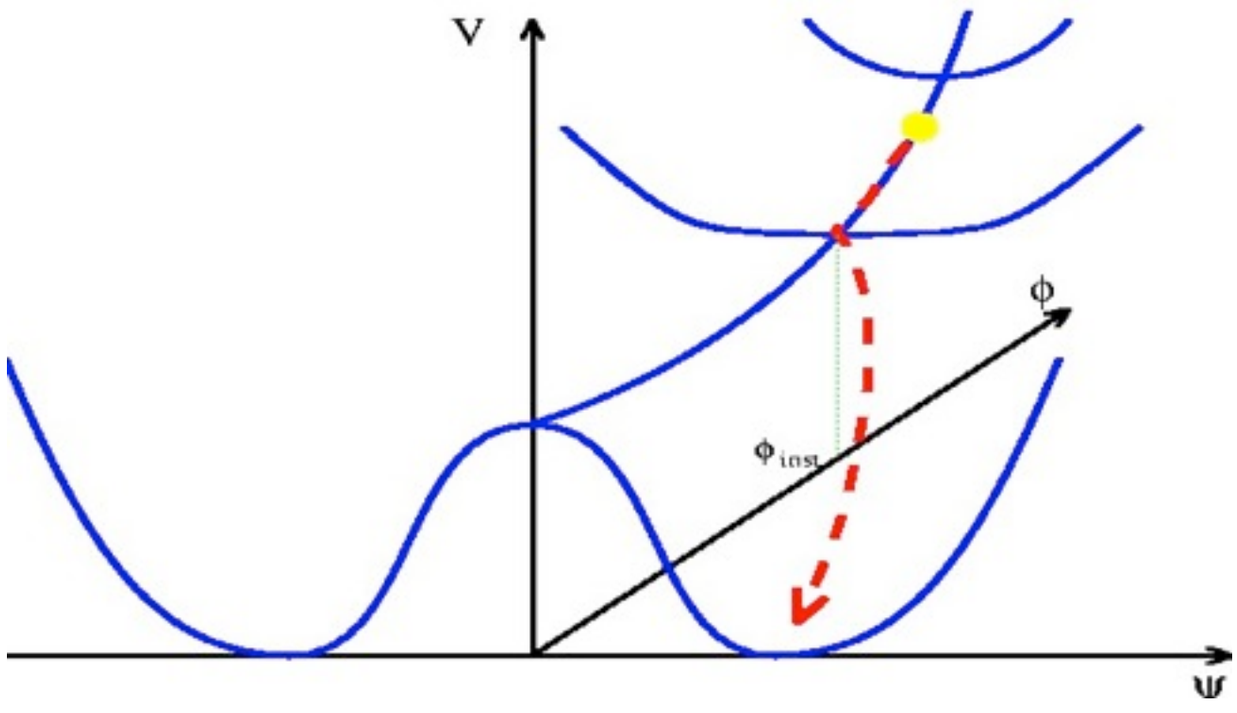


# Two field inflation – more general

$$V(\phi,\psi) = \frac{1}{2}m_\phi{}^2\phi^2 + \frac{1}{2}g^2\phi^2|\chi|^2 + \frac{1}{4}\lambda\left(|\chi|^2 - \frac{m^2}{\lambda}\right)^2$$

Found in SUSY models.

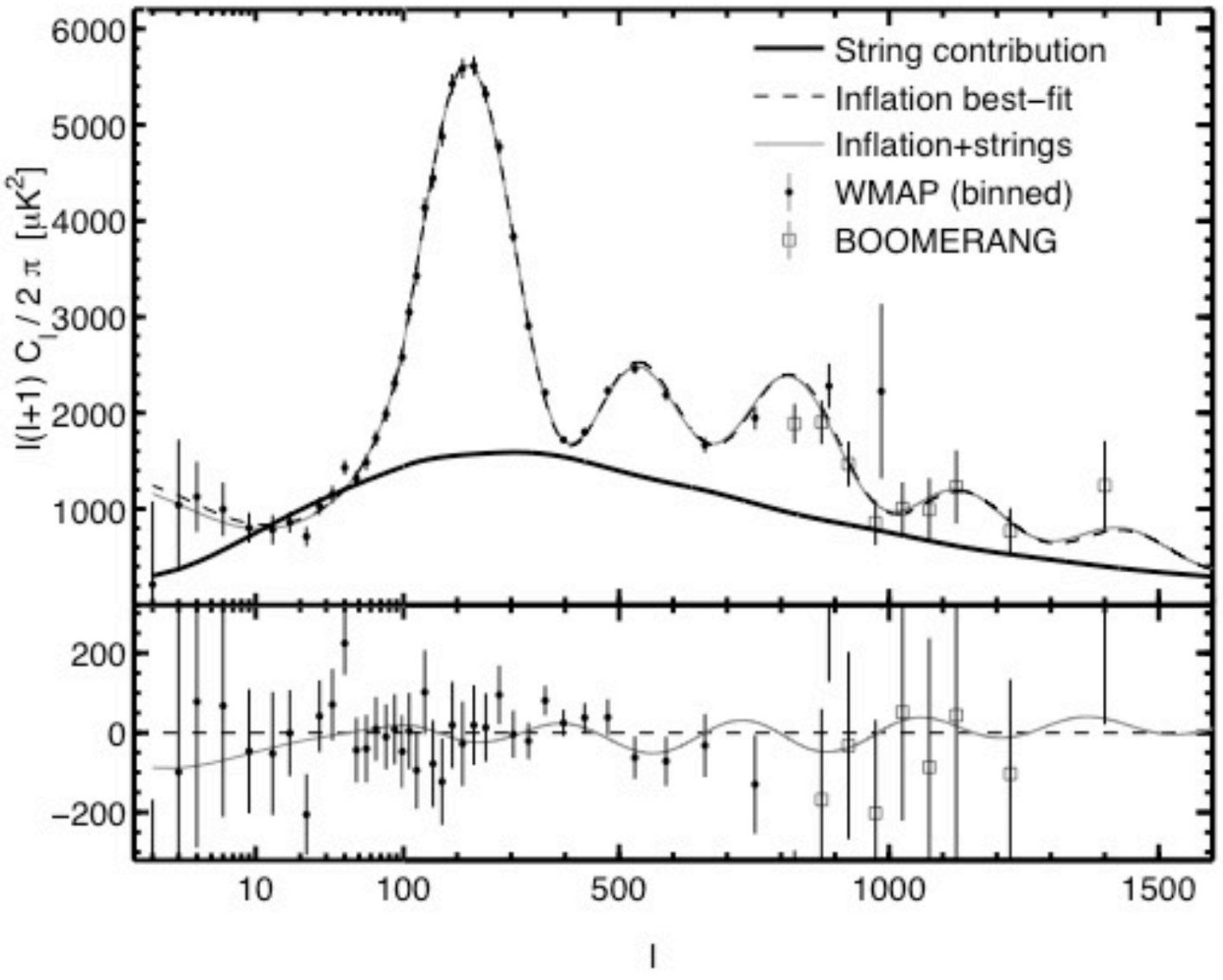
Better chance of success, plus lots of additional features, inc defect formation, ewk baryogenesis.




Inflation ends by triggering phase transition in second field.

Example of Brane inflation

Cosmic strings - may not do the full job but they can still contribute




Hybrid Inflation type models

String contribution < 11% implies $G\mu < 0.7 * 10^{-6}$.

Bevis et al 2007,2010.

Inflation model building today -- big industry

Multi-field inflation

Inflation in string theory and braneworlds

Inflation in extensions of the standard model

Cosmic strings formed at the end of inflation

The idea is clear though:

Use a combination of data (CMB, LSS, SN, BAO ...) to try and constrain models of the early universe through to models explaining the nature of dark energy today.

## Inflation in string theory -- non trivial

## The η problem in Supergravity -- N=1 SUGR Lagrangian:

$$\mathcal{L} = -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} + V_F, \quad \textbf{with} \quad V_F = e^{K/M_p^2}\left[K^{\varphi\bar{\varphi}}D_\varphi W\overline{D_\varphi W} - \frac{3}{M_p^2}|W|^2\right]$$

**and** $D_\varphi = \partial_\varphi W + \frac{1}{M_p^2}\partial_\varphi K \qquad K(\varphi,\bar{\varphi}) = K_0 + K_{\varphi\bar{\varphi}}\varphi\bar{\varphi} + \ldots$

**Expand K about φ=0**

$$\begin{aligned}\mathcal{L} &\approx -K_{\varphi\bar{\varphi}}\partial\varphi\partial\bar{\varphi} - V_0\left(1 + K_{\varphi\bar{\varphi}}|_{\varphi=0}\frac{\varphi\bar{\varphi}}{M_p^2} + \ldots\right) \\ &= -\partial\phi\partial\bar{\phi} - V_0\left(1 + \frac{\phi\phi}{M_p^2} + \ldots\right),\end{aligned}$$

**Canonically norm fields ϕ**

**Have model indep terms which lead to contribution to slow roll parameter η of order unity**

$$\Delta\eta = M_p^2\frac{\Delta V''}{V_0} = 1.$$

**So, need to cancel this generic term possibly through additional model dependent terms.**

**Ex 1: Warped D3-brane D3-antibrane inflation where model dependent corrections to V can cancel model indep contributions**

**[Kachru et al (03) -- KLMMT].**

**Find:**

$$V(\phi) = V_0(\phi) + \beta H^2 \phi^2$$

**β relates to the coupling of warped throat to compact CY space. Can be fine tuned to avoid η problem**

**Ex 2: DBI inflation -- simple -- it isn't slow roll as the two branes approach each other so no η problem**

**Ex 3: Kahler Moduli Inflation [Conlon & Quevedo 05]**

**Inflaton is one of Kahler moduli in Type IIB flux compactification. Inflation proceeds by reducing the F-term energy. No η problem because of presence of a symmetry, an almost no-scale property of the Kahler potential.**

$$V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n \tau_n}}{\mathcal{V}^2},$$

**Inflaton moduli: $\tau_n$**

$$V_{inf} = V_0 - \frac{4\tau_n W_0 a_n A_n e^{-a_n \tau_n}}{\mathcal{V}^2},$$

**Find:**
$$\begin{aligned} 0.960 &< n < 0.967, \\ -0.0006 &< \tfrac{dn}{d\ln k} < -0.0008, \\ 0 &< |r| < 10^{-10}, \end{aligned}$$

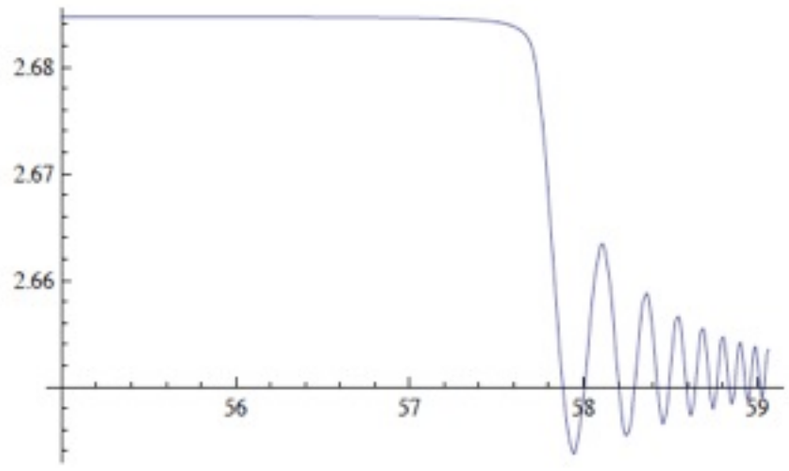
**with large volume modulus**

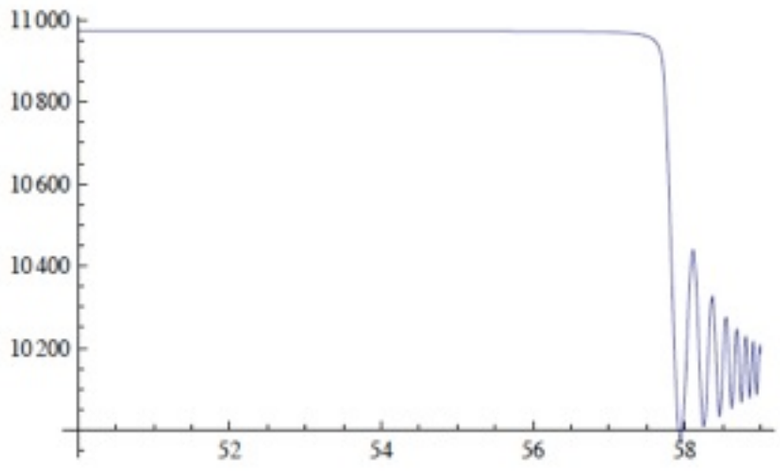
$$10^5 l_s^6 \leq \mathcal{V} \leq 10^7 l_s^6,$$

**and**
$$\begin{aligned} \eta &\approx -\frac{1}{N_e}, \\ \epsilon &< 10^{-12}, \end{aligned}$$

**for $N_e \approx$50-60 efolds with low energy scale**

$$V_{inf} \sim 10^{13}\text{GeV}.$$




**Inflaton** **[Blanco-Pillado et al 09]** **Volume modulus**

**Can include curvaton as second evolving moduli --** **Burgess et al 2010**

## Key inflationary parameters:

n: Perhaps Planck will finally determine whether it is unity or not.

r: Tensor-to-scalar ratio : considered as a smoking gun for inflation but also produced by defects and some inflation models produce very little.

dn/dln k : Running of the spectral index, usually very small -- probably too small for detection.

$f_{NL}$: Measure of cosmic non-gaussianity. Still consistent with zero, but tentative evidence of a non-zero signal in WMAP data which would provide an important piece of extra information to constrain models. For example, it could rule out single field models -- lots of current interest.

Gμ: string tension in Hybrid models where defects produced at end of period of inflation.

Also new perturbation generation mechanisms (e.g. Curvaton)

Perturbations not from inflaton but from extra field and then couple through to curvature perturbation

# Things not explored - no time

1. Gravitational waves from pre-heating

2. Non-Gaussianity from multi-field inflation

3. Nature of perturbations (adiabatic v non-adiabatic)

4. Thermal inflation and warm inflation

5. Going beyond slow roll

6. Inflation model building -- how easy in string theory.

7. Where is the inflaton in particle physics ? How fine tuned is it?

8. Low energy inflation (i.e. TeV scale).

9. Singularity -- eternal inflation !

10. Impact of multiverse on inflation.

11. Alternatives: pre-big bang, cyclic/ekpyrotic, string cosmology, varying speed of light, quantum gravity ....

# And so where are we today?

- Exciting time in cosmology -- Big Bang huge success.
- String - theory suggests we can consistently include gravity into particle physics.
- What started the big bang ?
- How did inflation emerge – if at all ?
- How did the spacetime dimensions split up?
- Where did the particle masses come from?
- Why are there just three families of particles?
- Why is the Universe accelerating today?
- What is the dark matter
- Where is all the anti-matter?

**Thank you for listening and good luck to you all with your research.**