Symmetries in particle physics

- Symmetries: Spins and their addition
- The eightfold way revisited
- Discrete symmetries:
  Charge conjugation, parity and time reversal

Symmetries
Spin-1/2 systems

Some generalities

- Spin-1/2 systems are often studied in physics.
  Example: electron and its spin, isospin, …

- Spin-statistics theorem suggests that such systems are fermionic in nature.

- Interesting in the context of this lecture:
  Basic building blocks of matter (quarks & leptons) are spin-1/2.

- Simple representation:

$$|\uparrow\rangle = |s = 1/2, s_z = 1/2\rangle\ , \quad |\downarrow\rangle = |s = 1/2, s_z = -1/2\rangle$$

Symmetries

Adding two spin-1/2

- Often, it is important to add spins
  Examples: bound states of spin-1/2 fermions, spin-orbit coupling, etc..
- If two spin-1/2 systems are added, the following objects can emerge: $|\uparrow\uparrow\rangle$, $|\uparrow\downarrow\rangle$, $|\downarrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$
- Naively, they have spin 1, 0, or -1, respectively.
- But: Need to distinguish total spin s and its projection onto the "measurement axis" s_z
  (here, z has been chosen for simplicity)

Symmetries

Adding two spin-1/2

- Then the truly relevant states are for s=1 (triplet)

$$|\uparrow\uparrow\rangle = |s=1, s_z=1\rangle \; ,$$
$$\frac{1}{\sqrt{2}}|(\uparrow\downarrow + \downarrow\uparrow)\rangle = |s=1, s_z=0\rangle \; ,$$
$$|\downarrow\downarrow\rangle = |s=1, s_z=-1\rangle$$

and for s=0 (singlet)

$$\frac{1}{\sqrt{2}}|(\uparrow\downarrow - \downarrow\uparrow)\rangle = |s=0, s_z=0\rangle$$

- Note: the triplets are symmetric, the singlet is anti-symmetric. Catchy: $2 \otimes \bar{2} = 3 \oplus 1$

Symmetries

Clebsch-Gordan coefficients

- The coefficients in front of the new $|s, s_z\rangle$ states can be calculated (or looked up). They go under the name of Clebsch-Gordan coefficients.
- Formally speaking, they are defined as follows:

$$\langle s^1, s_z^1; s^2, s_z^2 | s^1, s^2; s, s_z \rangle$$

indicating that two spin systems s^1 and s^2 are added to form a new spin system with total spin s. Obviously, it is not only the total spin of each system that counts here, but also its orientation. This is typically indicated through a "magnetic" quantum number, m, replacing s_z in the literature.

Symmetries

Clebsch-Gordan coefficients

Special cases::

- For s=0:

$$\langle s^1, m^1; s^2, m^2 | s^1, s^2; 0, 0\rangle = \delta_{s^1,s^2}\delta_{m^1,-m^2}\frac{(-1)^{s^1-m^1}}{\sqrt{2s_2+1}}$$

- Clebsch-Gordan tables for two cases:

  (a square-root over each coefficient is implied)

Notation:

m_1	m_2	J M	J M	...
m_1	m_2	Coefficients		
m_1	m_2			
⋮	⋮			

1/2×1/2

m_1	m_2	1, +1	1, 0	0, 0	1, −1
+1/2	+1/2	1			
+1/2	−1/2		1/2	1/2	
−1/2	+1/2		1/2	−1/2	
−1/2	−1/2				1

1×1/2

m_1	m_2	3/2, +3/2	3/2, +1/2	1/2, +1/2	3/2, −1/2	1/2, −1/2	3/2, −3/2
+1	+1/2	1					
+1	−1/2		1/3	2/3			
0	+1/2		2/3	−1/3			
0	−1/2				2/3	1/3	
−1	+1/2				1/3	−2/3	
−1	−1/2						1

Symmetries
From spin to isospin

Who carries isospin?

- Remember Heisenberg's proposal:
  p and n are just two manifestations of the same particle, the nucleon. Identify them with the isospin-up and isospin-down states of the nucleon:

$$|p\rangle = |1/2, 1/2\rangle\,, \quad |n\rangle = |1/2, -1/2\rangle$$

- Catch: Isospin conserved in strong interactions!
- Will dwell on that a bit: Play with pions, nucleons and Delta's.

(Note: Multiplicity in each multiplet: 2I+1).

Symmetries

From spin to isospin

Dynamical implications: Bound states (deuteron)

- Add two nucleons: can have isosinglet and isotriplet.

$$|0,0\rangle = \frac{1}{\sqrt{2}}|pn - np\rangle$$

$$|1,1\rangle = |pp\rangle\ , \quad |1,0\rangle = \frac{1}{\sqrt{2}}|pn + np\rangle\ , \quad |1,-1\rangle = |nn\rangle$$

- No pp, nn-bound states ⇨ deuteron = isosinglet !!!
- Consider processes (+ their isospin amplitudes, below)

$$p + p \rightarrow d + \pi^+ \quad p + n \rightarrow d + \pi^0 \quad n + n \rightarrow d + \pi^- .$$

$$1 \qquad 1/\sqrt{2} \qquad 1$$

$$|1,1\rangle \rightarrow |1,1\rangle \quad \frac{1}{\sqrt{2}}\left(|1,0\rangle + |0,0\rangle\right) \rightarrow |1,0\rangle \quad |1,-1\rangle \rightarrow |1,-1\rangle$$

Symmetries

From spin to isospin

Who carries isospin?

- Nucleons in isospin notation:

$$|p\rangle = |\frac{1}{2}, \frac{1}{2}\rangle \,, \quad |n\rangle = |\frac{1}{2}, -\frac{1}{2}\rangle$$

- Pions in isospin notation:

$$|\pi^+\rangle = |1,1\rangle \,, \quad |\pi^0\rangle = |1,0\rangle \,, \quad |\pi^-\rangle = |1,-1\rangle$$

- Delta's in isospin notation:

$$|\Delta^{++}\rangle = |\frac{3}{2}, \frac{3}{2}\rangle \,, \quad |\Delta^{+}\rangle = |\frac{3}{2}, \frac{1}{2}\rangle \,,$$

$$|\Delta^{0}\rangle = |\frac{3}{2}, -\frac{1}{2}\rangle \,, \quad |\Delta^{-}\rangle = |\frac{3}{2}, -\frac{3}{2}\rangle$$

Symmetries

Isospin and scattering amplitudes

Use isospin for pion-nucleon scattering amplitudes

- Elastic processes

$$(a)\ \pi^+ + p \to \pi^+ + p \qquad (b)\ \pi^0 + p \to \pi^0 + p$$
$$(c)\ \pi^- + p \to \pi^- + p \qquad (d)\ \pi^+ + n \to \pi^+ + n$$
$$(e)\ \pi^0 + n \to \pi^0 + n \qquad (f)\ \pi^- + n \to \pi^- + n$$

- Charge exchange processes

$$(g)\ \pi^+ + n \to \pi^0 + p \qquad (h)\ \pi^0 + p \to \pi^+ + n$$
$$(i)\ \pi^0 + n \to \pi^- + p \qquad (j)\ \pi^- + p \to \pi^0 + n$$

Symmetries

Isospin and scattering amplitudes

Use isospin for pion-nucleon scattering amplitudes

- Remember: pions and nucleons are isopin-1 and 1/2. The total isospin is either 1/2 or 3/2 and thus, there are only two independent amplitudes: $\mathcal{M}_3$ and $\mathcal{M}_1$
- Use Clebsch-Gordan coefficients:

$$\pi^+ + p = \left|\frac{3}{2}, \frac{3}{2}\right\rangle \qquad \pi^+ + n = \frac{1}{\sqrt{3}}\left|\frac{3}{2}, \frac{1}{2}\right\rangle + \sqrt{\frac{2}{3}}\left|\frac{1}{2}, \frac{1}{2}\right\rangle$$

$$\pi^0 + p = \sqrt{\frac{2}{3}}\left|\frac{3}{2}, \frac{1}{2}\right\rangle - \frac{1}{\sqrt{3}}\left|\frac{1}{2}, \frac{1}{2}\right\rangle \qquad \pi^0 + n = \sqrt{\frac{2}{3}}\left|\frac{3}{2}, -\frac{1}{2}\right\rangle + \frac{1}{\sqrt{3}}\left|\frac{1}{2}, -\frac{1}{2}\right\rangle$$

$$\pi^- + p = \frac{1}{\sqrt{3}}\left|\frac{3}{2}, -\frac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\left|\frac{1}{2}, -\frac{1}{2}\right\rangle \qquad \pi^- + n = \left|\frac{3}{2}, -\frac{3}{2}\right\rangle$$

Symmetries

Isospin and scattering amplitudes

Use isospin for pion-nucleon scattering amplitudes

- Then: Reactions (a) and (f) are pure 3/2:

$$(a)\ \pi^+ + p \to \pi^+ + p$$
$$(f)\ \pi^- + n \to \pi^- + n$$

$$\mathcal{M}_a = \mathcal{M}_f = \mathcal{M}_3$$

- Other reactions are mixtures (coefficients given by the Clebsch-Gordans), e.g.

$$(c)\ \pi^- + p \to \pi^- + p \qquad \mathcal{M}_c = 1/3\mathcal{M}_3 + 2/3\mathcal{M}_1$$
$$(j)\ \pi^- + p \to \pi^0 + n \qquad \mathcal{M}_j = \sqrt{2}/3\mathcal{M}_3 - \sqrt{2}/3\mathcal{M}_1$$

Symmetries

Isospin and scattering amplitudes

Use isospin for pion-nucleon scattering amplitudes

- Therefore, the cross sections behave like

$$\sigma_a : \sigma_c : \sigma_j = 9|\mathcal{M}_3|^2 : |\mathcal{M}_3 + 2\mathcal{M}_1|^2 : 2|\mathcal{M}_3 - \mathcal{M}_1|^2$$

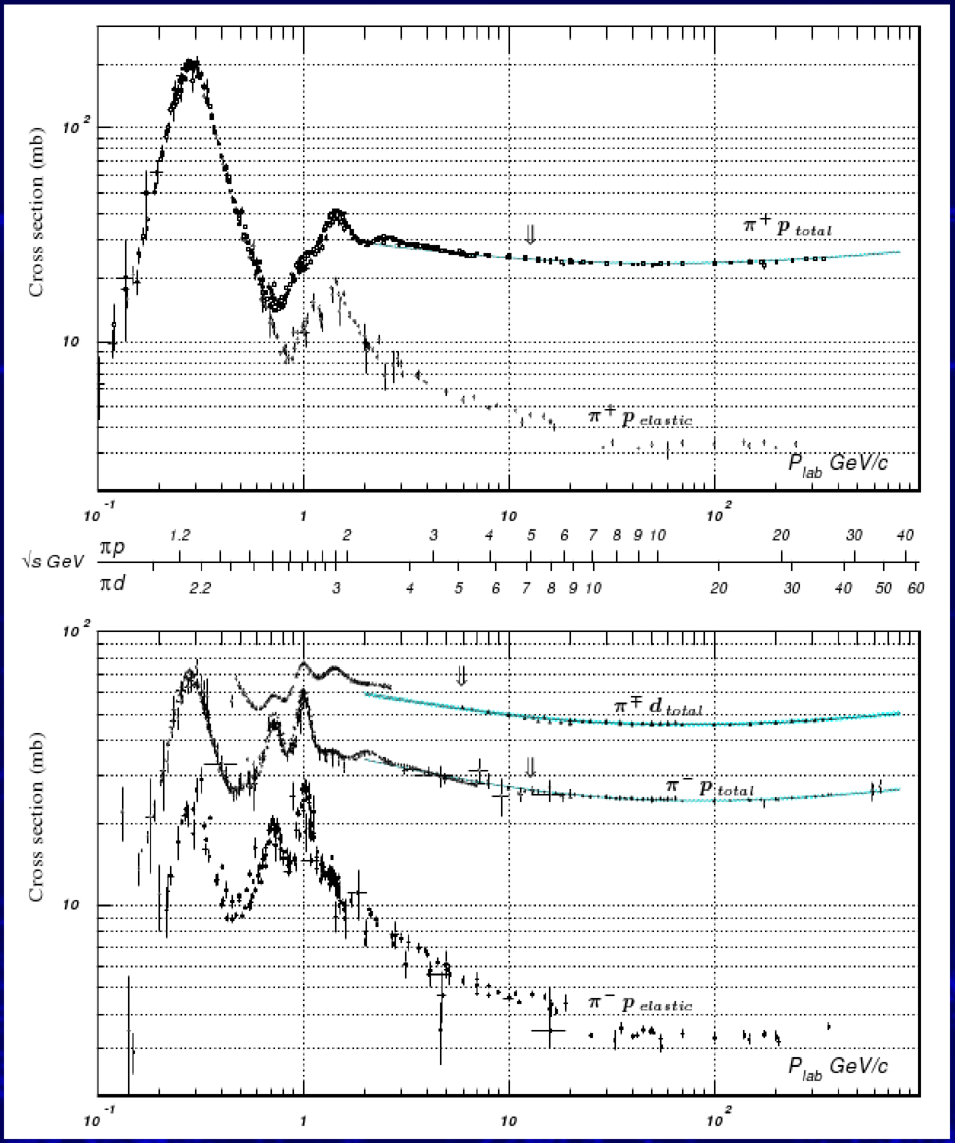
- At a c.m. energy of 1232 MeV there is a dramatic bump in the pion-nucleon scattering cross section, first discovered by Fermi in 1951. There, the pion and nucleon form a short-lived resonance state, the Δ which we know to carry I = 3/2.

Symmetries

Isospin and scattering amplitudes

- At c.m. energies around the Δ-mass, one can expect that $\mathcal{M}_3 \gg \mathcal{M}_1$, and therefore, there $\sigma_a : \sigma_c : \sigma_j \approx 9 : 1 : 2$
- Experimentally, it is simpler to combine (c) and (j), leading to

$$\frac{\sigma_{\rm tot}(\pi^+ + p)}{\sigma_{\rm tot}(\pi^- + p)} = 3$$



Symmetries
Isospin and G-parity

Pions and isospin: G-parity

- How does this work for the mesons (the pions) ?
- Pions = bound states of a quark and an antiquark, so naively: "Just add the isospins like the spins".
- But: Rules of spin addition not sufficient. How to "bar" a spin? Problem: want to preserve some symmetries like charge conjugation under "barring".
- G-parity (a group-theory construct) demands:

$$\hat{G}|\pi\rangle = -|\pi\rangle \ , \quad \hat{G}|n\pi\rangle = (-1)^n |n\pi\rangle$$

conserved quantum number in strong interactions.

Symmetries

Isospin and G-parity

Pions and isospin: G-parity

- Altogether: The pion (isospin=1) multiplet reads

$$|\pi^+\rangle = |u\bar{d}\rangle\,,\ \ \pi^0\rangle = \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle\,,\ \ \pi^-\rangle = |d\bar{u}\rangle\,.$$

- The unexpected minus-sign in the neutral pion (compare with spin) is due to the G-parity acting on the quarks and anti-quarks (the former have positive, the latter negative G-parity).

The eightfold way, revisited
Some SU(3) relations

Why SU(3)?

- In isospin, there are two quarks related by symmetry, $|u\rangle = |1/2, 1/2\rangle$ and $|d\rangle = |1/2, -1/2\rangle$
- The group related to this is the spin group, or SU(2). Its generators are the Pauli matrices,

$$\sigma_{1,2,3} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- The pions can be identified with σ_3 and the two linear combinations (of definite charge)

$$\sigma_\pm = \frac{1}{\sqrt{2}}(\sigma_1 \pm i\sigma_2)$$

The eightfold way, revisited
Some SU(3) relations

Why SU(3)?

- For three states $|u\rangle, |d\rangle, |s\rangle$ similarly related through a symmetry, one could think about the group SU(3).
- Its generators are the Gell-Mann matrices.
- In SU(3), the mesons can be connected to suitable linear combinations of the Gell-Mann matrices (see next slide)
- Note: QCD's gauge group is also SU(3). differentiate between SU(3) of flavour (up, down, strange) and SU(3) of colour (red, green, blue), although group theory is the same!

The eightfold way, revisited

Some SU(3) relations

The Gell-Mann matrices

$$\lambda_{1,2,3} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_{4,5,6} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_{7,8} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The eightfold way, revisited
Some SU(3) relations

Singlet-octet mixing

- Note: In the meson sector, also a "singlet meson" bit contributes, with a wave function of the form

$$|\psi_1\rangle = \frac{1}{\sqrt{3}}\left(|u\bar{u}\rangle + |d\bar{d}\rangle + |s\bar{s}\rangle\right)$$

  It could be realised through a unit matrix.

- Typically there is a mixing with octet wave functions, most notably examples are the $\eta - \eta'$ and the $\omega - \phi$ mixing in the pseudoscalar and vector multiplet. So, typically, there are nine mesons per SU(3)-multiplet.

The eightfold way, revisited

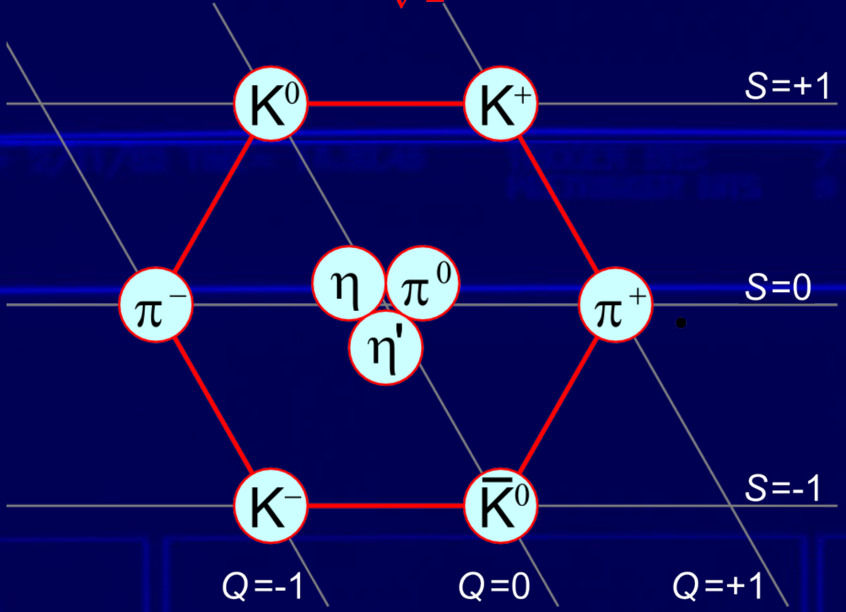
The pseudoscalar mesons

Here, the spins anti-align $\frac{1}{\sqrt{2}}|\uparrow\downarrow - \downarrow\uparrow\rangle$

$$|K^0\rangle = |d\bar{s}\rangle \quad |K^+\rangle = |u\bar{s}\rangle$$

$$|\pi^-\rangle = |d\bar{u}\rangle \quad |\pi^+\rangle = |u\bar{d}\rangle$$

$$|K^-\rangle = |s\bar{u}\rangle \quad |\bar{K}^0\rangle = |s\bar{d}\rangle$$



$$|\pi^0\rangle = \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle$$

$$|\eta\rangle = \frac{\cos\theta}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle + \frac{\sin\theta}{\sqrt{3}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle$$

$$|\eta'\rangle = -\frac{\sin\theta}{\sqrt{3}}|u\bar{u} + d\bar{d} + s\bar{s}\rangle + \frac{\cos\theta}{\sqrt{3}}|u\bar{u} + d\bar{d} - 2s\bar{s}\rangle$$

The eightfold way, revisited

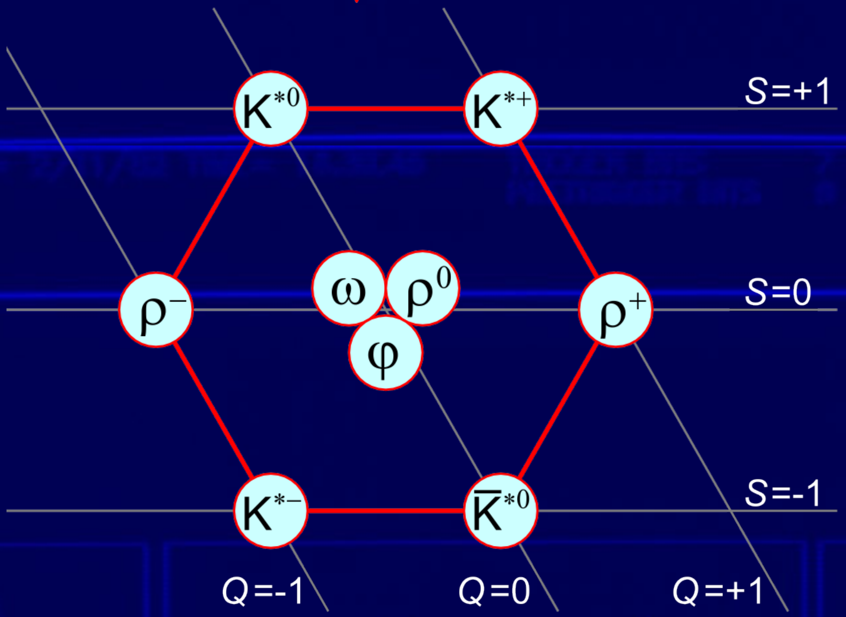
The vector mesons

Here, the spins align $|\uparrow\uparrow\rangle, \frac{1}{\sqrt{2}}|\uparrow\downarrow + \downarrow\uparrow\rangle, |\downarrow\downarrow\rangle$

$|K^{*0}\rangle = |d\bar{s}\rangle \quad |K^{*+}\rangle = |u\bar{s}\rangle$

$|\rho^{-}\rangle = |d\bar{u}\rangle \quad |\rho^{+}\rangle = |u\bar{d}\rangle$

$|K^{*-}\rangle = |s\bar{u}\rangle \quad |\bar{K}^{*0}\rangle = |s\bar{d}\rangle$



$$|\rho^{0}\rangle = \frac{1}{\sqrt{2}}|u\bar{u} - d\bar{d}\rangle \qquad |\omega\rangle = \frac{1}{\sqrt{2}}|u\bar{u} + d\bar{d}\rangle \qquad |\phi\rangle = |s\bar{s}\rangle$$

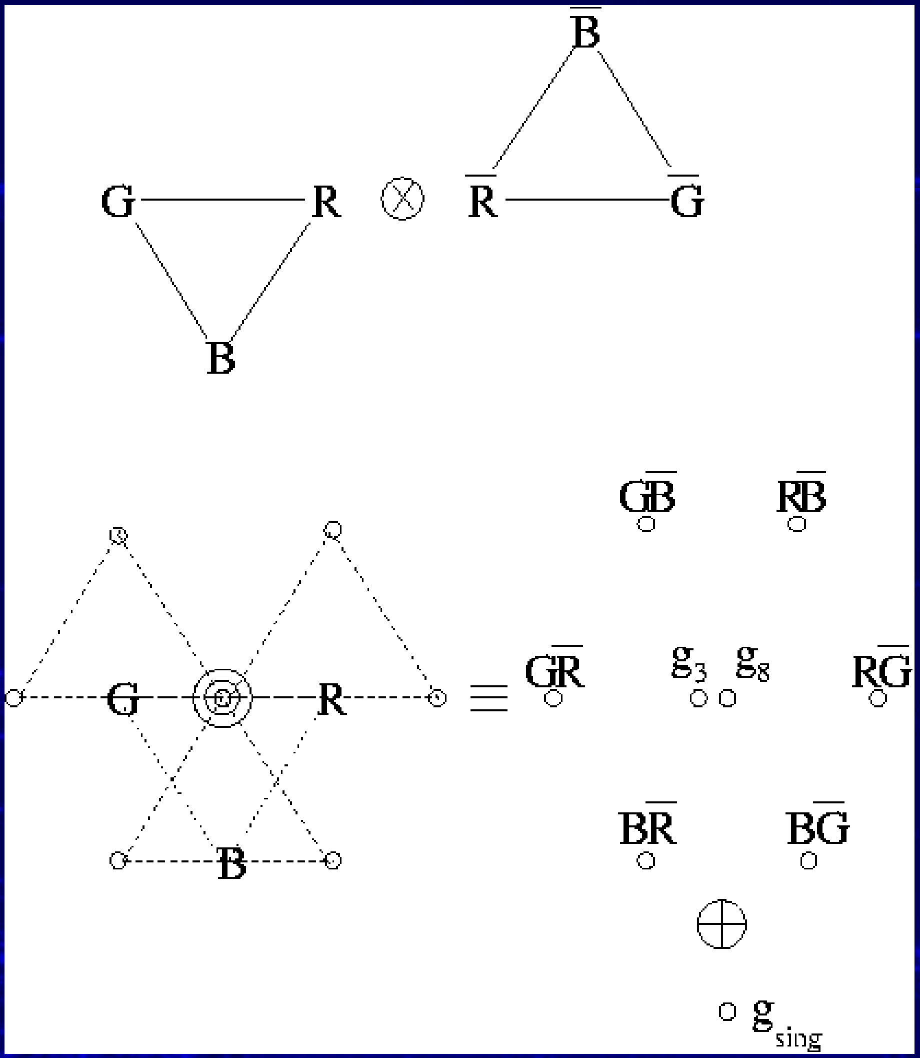
The eightfold way, revisited

An alternative look

Example for colour:

Basic equation: $3 \otimes \bar{3} = 8 \oplus 1$

- This should explain the eight gluons and the absence of the ninth (the singlet) one.
- Similar for the mesons, just replace colour with flavour.



Discrete symmetries: Some examples

Charge conjugation, parity, time reversal

- In addition to continuous symmetries, which can reflect properties of space-time (like, e.g. under rotations, boosts, etc.) or of dynamics (gauge symmetries), there may also be discrete symmetries.
- Most important examples:
  - Charge conjugation: ALL charges (electric, colour, etc.) are inverted. Operator for that: $\hat{C}$
  - Parity: Move from a left-handed coordinate system to a right-handed one (mirror). Realised through $\hat{P}$
  - Time reversal: Invert time axis, operator: $\hat{T}$

Discrete symmetries:

Parity and time reversal

- The operators of the discrete symmetries related to space time, $\hat{P}$ and $\hat{T}$, are quite obvious when acting on four-vectors:

$$\hat{P}^{\mu}_{\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}, \quad \hat{P}^{\mu}_{\nu} x^{\nu} = \tilde{x}^{\mu} = (t, -\vec{x})$$

$$\hat{T}^{\mu}_{\nu} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad \hat{T}^{\mu}_{\nu} x^{\nu} = \tilde{x}^{\mu} = (-t, \vec{x})$$

- Note: Although at first it looks similar, the parity operator is not the metric! (position of the indices).

Discrete symmetries:
Parity

Parity violation

- Until the 1950's people believed that parity (symmetry under "mirroring") was conserved, and there have been many tests in electromagnetic and strong interactions but none in weak interactions.
- This led Lee and Yang to propose an experiment, which was later that year carries out by C.S.Wu.
- In this experiment radioactive ^{60}Co was carefully aligned such that its spin would point into the direction of the positive z-axis.

Discrete symmetries:
Parity

Parity violation

- Then the Cobalt undergoes a β-decay, emitting an electron and an anti-neutrino. Wu found that most of the electrons would be emitted into the positive z-direction.
- If the process is "mirrored", the spin of the nucleus points along the negative z-axis, but the electrons would still be emitted into the positive z-direction.
- This different behaviour is called axial-vector and vector for spin and momentum (and respective similar quantities).

Discrete symmetries: Parity

Helicity, chirality, and all that

- Note: Reflections turn left-handed coordinate systems into right-handed ones and vice versa, this affects spins etc.
- More physical definition: Define handedness as spin with respect to the axis of motion (technically speaking, this is helicity). For massive particles this is not Lorentz-invariant, but for massless ones it is.
- Therefore, helicity is a meaningful, fixed property of massless particles, called chirality.

Discrete symmetries: <u>Parity</u>

Helicity, chirality, and all that

- In the Standard Model:
  ALL NEUTRINOS ARE LEFT-HANDED
- This can be seen by considering pion decays into muon + antineutrino. In the rest frame of the pion, the muon and neutrino come out back-to-back and the spins have to add to 0 (since the pion has spin-0). Therefore, the handedness (spin-direction) of the anti-neutrino equals the handedness of the muon.
- In this experiment, up to now, muons with only one helicity/handedness have been found.

Discrete symmetries: CP violation

Charge conjugation, parity, time reversal

- For a long time, it was thought that ALL laws of nature on the particle level are invariant under each of these three symmetries.
- But: While this is true for QED and QCD, the weak interactions proved to be maximally parity violating (only left-handed neutrinos)!!! In addition, the weak interactions show small violation of the combined $\hat{C}\hat{P}$ operation. When this was discovered, it came as a shock. Today we know that this is due to the complex phases in the CKM matrix.

Discrete symmetries: CP violation

Mixing in the system of the neutral mesons

- A prime example for $\hat{C}\hat{P}$ violation is in the system of the neutral mesons, like the neutral kaons.
- In terms of flavour, there are two eigenstates, $|K^0\rangle = |d\bar{s}\rangle$ and $|\bar{K}^0\rangle = |s\bar{d}\rangle$. However, experimentally, two states with wildly different lifetime are observed: $|K_S\rangle$ and $|K_L\rangle$, which predominantly decay into two or three pions, respectively. Therefore they have different CP eigenvalues.

Discrete symmetries:
CP violation

Mixing in the system of the neutral mesons (cont'd)

- These CP eigenstates are nearly perfect mixtures of the flavour eigenstates.

$$|K_\pm\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \mp |\bar{K}^0\rangle\right) \quad \hat{C}\hat{P}|K_\pm\rangle = \pm|K_\pm\rangle$$

$$|K_{S,L}\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_\pm\rangle + \varepsilon|\bar{K}_\mp\rangle\right)$$

- ε is related to the amount of CP-violation in the kaon system (the prob. for decays into the "wrong" number of pions).

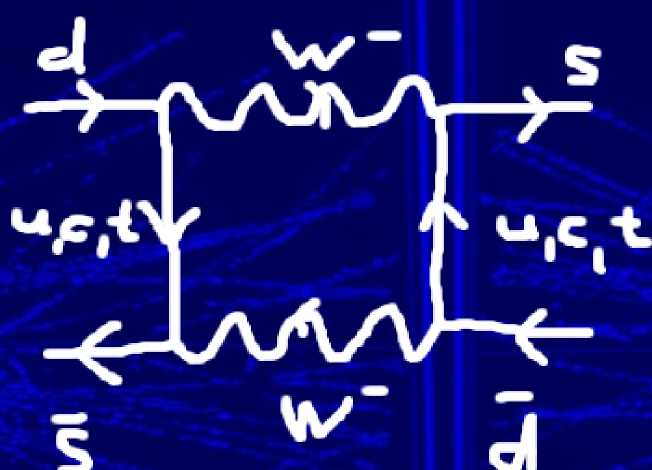
Discrete symmetries: CP violation

Mixing in the system of the neutral mesons (cont'd)

- ε is related to the Hamiltonian of the kaon system:

$$\left\langle \begin{matrix} K^0 \\ \bar{K}^0 \end{matrix} \middle| \hat{H} \middle| \begin{matrix} K^0 \\ \bar{K}^0 \end{matrix} \right\rangle$$

- The off-diagonal elements are given by amplitudes for the transition between the two kaons:



- The amplitude is proportional to a product of four CKM matrix elements of the form

$$V_{td}^* V_{ts}^* V_{td} V_{ts}$$

- This allows for complex values in the Hamiltonian matrix ⇨ CP violation.

Discrete symmetries:
The CPT-theorem

Charge conjugation, parity, time reversal

- Despite of CP violation, up to now, no violation of the combined version of all three discrete symmetries has been found. So CPT seems to be a true symmetry of the world.
- Ultimately, this allows for causal structures of the theory as realised up to now.