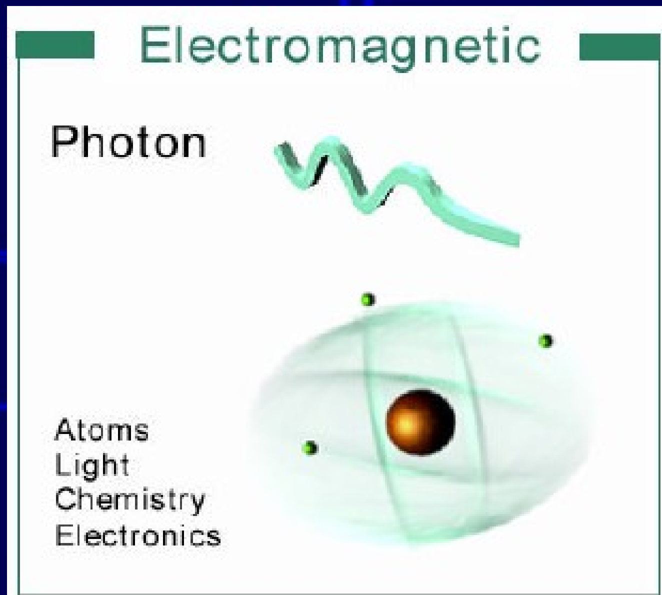
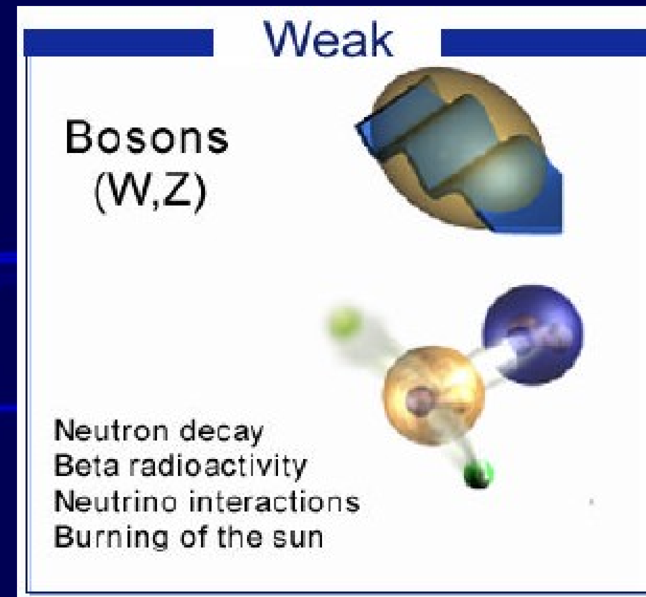
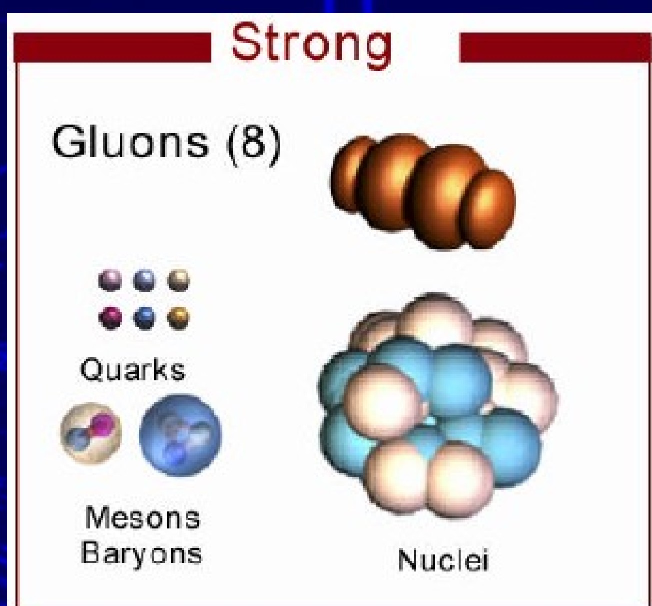
Forces and Interactions

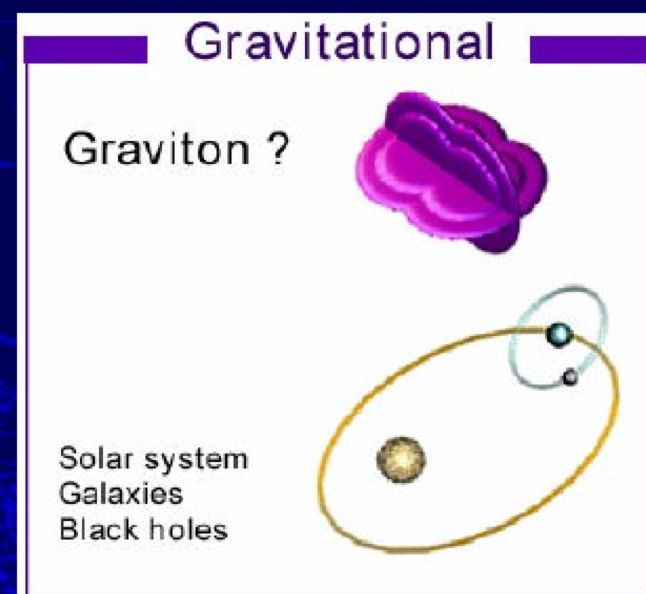
- The four forces in Nature
- Exchange particles mediate interactions
- Feynman diagrams:
  A graphical view into particle interactions
- QED: The mother of all gauge theories
- QCD: The theory of strong interactions
- Electroweak interactions and the Higgs boson
- The role of conservation laws

The four forces in Nature





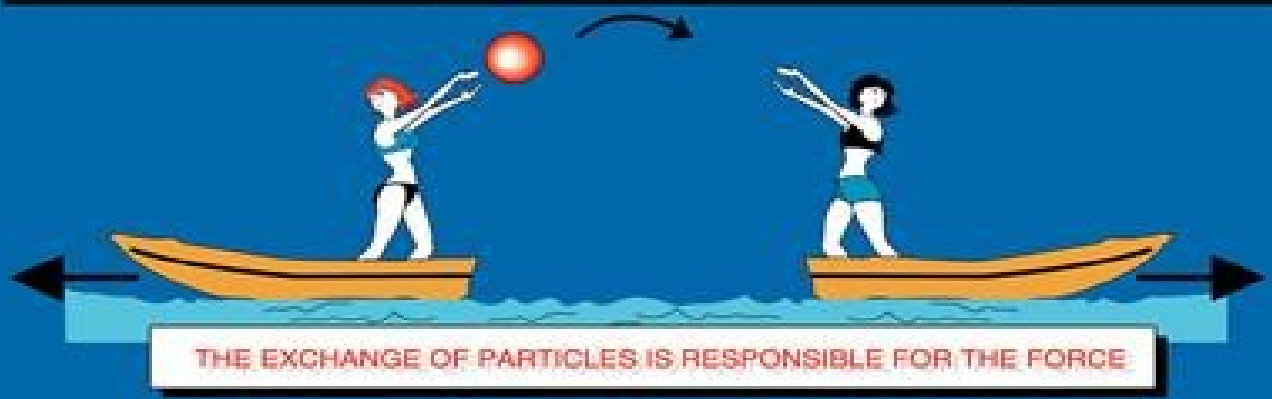




The four forces in Nature

The forces in Nature

TYPE	INTENSITY OF FORCES (DECREASING ORDER)	BINDING PARTICLE (FIELD QUANTUM)	OCCURS IN :
STRONG NUCLEAR FORCE	~ 1	GLUONS (NO MASS)	ATOMIC NUCLEUS
ELECTRO -MAGNETIC FORCE	~ 10^{-3}	PHOTONS (NO MASS)	ATOMIC SHELL ELECTROTECHNIQUE
WEAK NUCLEAR FORCE	~ 10^{-5}	BOSONS Z^0 , W+ , W- (HEAVY)	RADIOACTIVE BETA DESINTEGRATION
GRAVITATION	~ 10^{-38}	GRAVITONS (?)	HEAVENLY BODIES



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Exchange particles mediate interactions

- In classical physics: Forces due to potentials.
- Interaction due to coupling of objects with potentials. Forces can only indirectly be seen, e.g. through acceleration etc..
- New feature of quantum field theory: Quantising the interactions
  ⇨ particle interpretation
- Manifestation: light-by-light scattering (absent in classical electrodynamics, but present in quantum electrodynamics)

Feynman diagrams

Important:

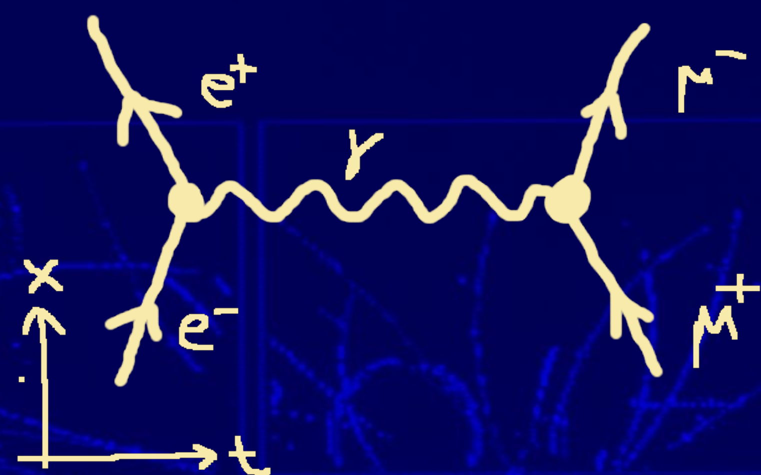
Feynman diagrams are NOT processes.

They are pictorial representations of terms in the perturbative expansion of quantum mechanical transition amplitudes. The expansion parameters are the coupling constants.

(Perturbative .expansion = Taylor series in small parameter – here, the small parameter is identified with the coupling)

Feynman diagrams

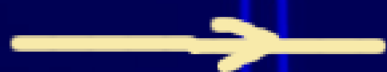
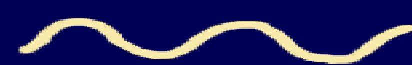
- Feynman diagrams can help to visualise certain aspects of scattering processes.
- Example: electron-positron annihilation into a muon pair. Remember the Stueckelberg-Feynman interpretation of antiparticles as going backwards in time. (the physical states are labelled though).

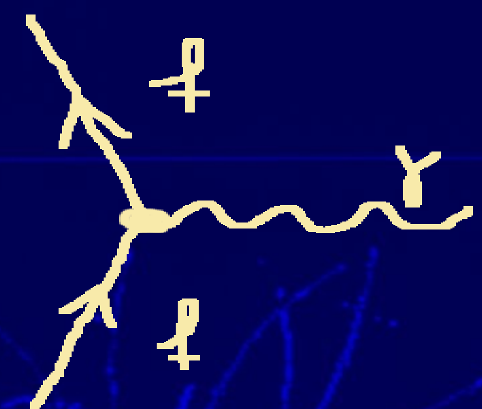


- Remember: In quantum mechanics, amplitudes are squared ⇨ so are Feynman diagrams

Electromagnetic interactions: QED

- Quantum Electro Dynamics (QED) is the oldest, simplest and most successful of all dynamics theories
- Building blocks: propagators of the particles (arrows and wavy lines for fermions and photons)



and vertices of fermions (f) and photons (γ).

- Vertex ~ ee_f with e_f is partial charge of the fermion and e is related to the fine structure constant $\alpha = e^2/(4\pi) \approx 1/137$

Electromagnetic interactions: QED

- To describe more complicated processes, building blocks are patched together.
- Example: e^-e^--scattering (Moller-scattering)

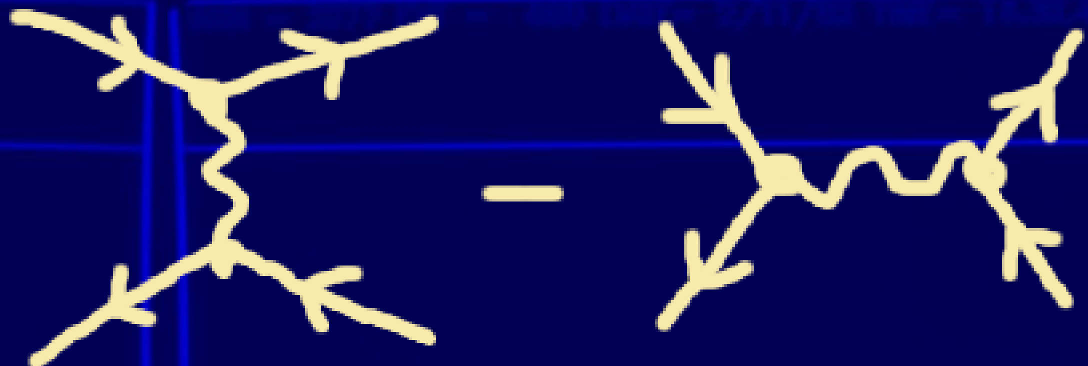
two diagrams with "-" sign (identical fermions)
⇨ quantum mechanics cannot decide which outgoing electron ends where.

Electromagnetic interactions: QED

- Remember: Feynman diagrams purely symbolic, assignment of time and space axes arbitrary – vertical and horizontal distances in Feynman diagrams have nothing to do with physical separations.
- Processes are defined by external ("real") particles. Internal particles are called "virtual", since they cannot be directly observed. Any attempt to observe them directly changes the process entirely.
- Feynman diagrams are complex numbers, contributing to the full QM transition amplitude.

Electromagnetic interactions: QED

Example: e^-e^+-scattering (Bhabha-scattering)

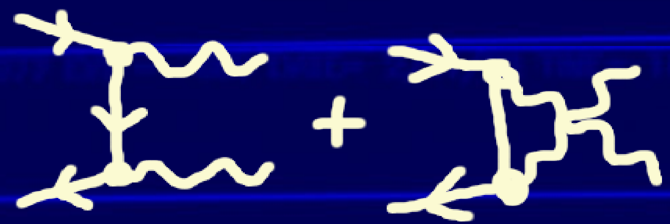


- Note: diagrams quite similar – in fact their squares have identical signs.
- Connection to Coulomb repulsion and attraction subtle – bound states through exchange of more photons (higher orders).

Electromagnetic interactions: QED

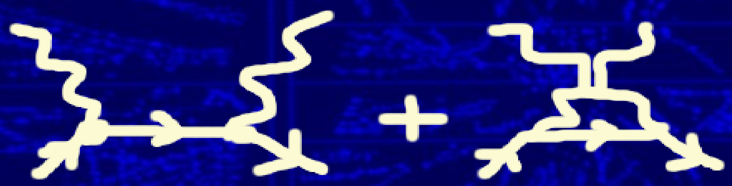
Further examples:

- pair-annihilation



- pair-creation,

- and Coulomb-scattering



Electromagnetic interactions: QED

Higher order corrections (to Bhabha scattering)

- Lead to divergent expressions
  - ⇨ must be renormalised (tricky procedure)
  - ⇨ couplings not constants, "run" (scale dependence!!!)

Electromagnetic interactions: QED

New processes: light-by-light scattering

- exploits quantum character of photons

Electromagnetic interactions: QED

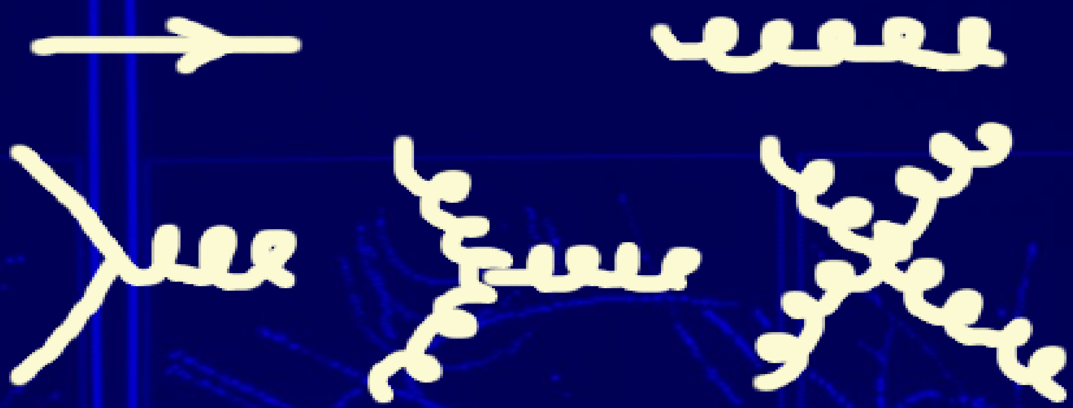
- Most precisely tested theory.
  Lighthouse example: magnetic moment (g=2)
  anomalous magnetic moment of the electron $a = \frac{g-2}{2}$
- History started with Schwinger, 1948:
  First loop calculation ever: $a = \alpha/(2\pi) \approx 0.0011614$
- By now: calculated up to order α^4:

$$a = 0.00115965218085(76)(\text{measured})$$

⇨ most precise value for $\alpha^{-1} = 137.035999710(96)$

Strong interactions: QCD

- In contrast to QED (charged fermions, neutral photons), the gluons carry a colour-charge
  ⇨ self-interactions of the gluons
- Therefore the basic building blocks are:



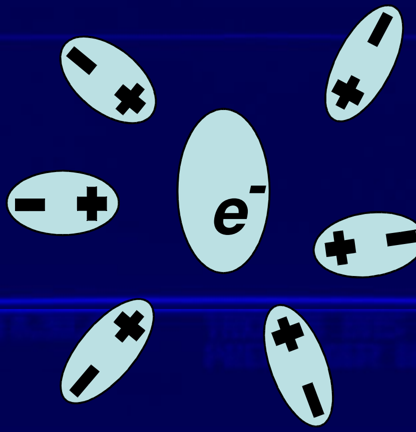
- Quark-quark-gluon and threee-gluon vertex $\sim g_s$, four-gluon vertex $\sim g_s^2$.

Strong interactions: QCD

Something about colour:

- Quarks come in three colours (r, g, b), gluons alter the colour of the quark in the interaction (vertex)
  ➪ gluons carry both a colour and an anti-colour
  ➪ naively: 9 gluons.
- But: There are eight gluons only!
- Reason: The singlet ($r\bar{r}+g\bar{g}+b\bar{b}$) has identical colour structure – no colour change - with photon
  ➪ cannot belong to the structure of QCD.

Strong interactions: QCD



Charge screening in QED:

- QED: Charge screening reduces effective charge seen by test-charge.
- By probing deeper, the "virtual cloud" of electron-positron pairs is penetrated ⇨ charge increases.

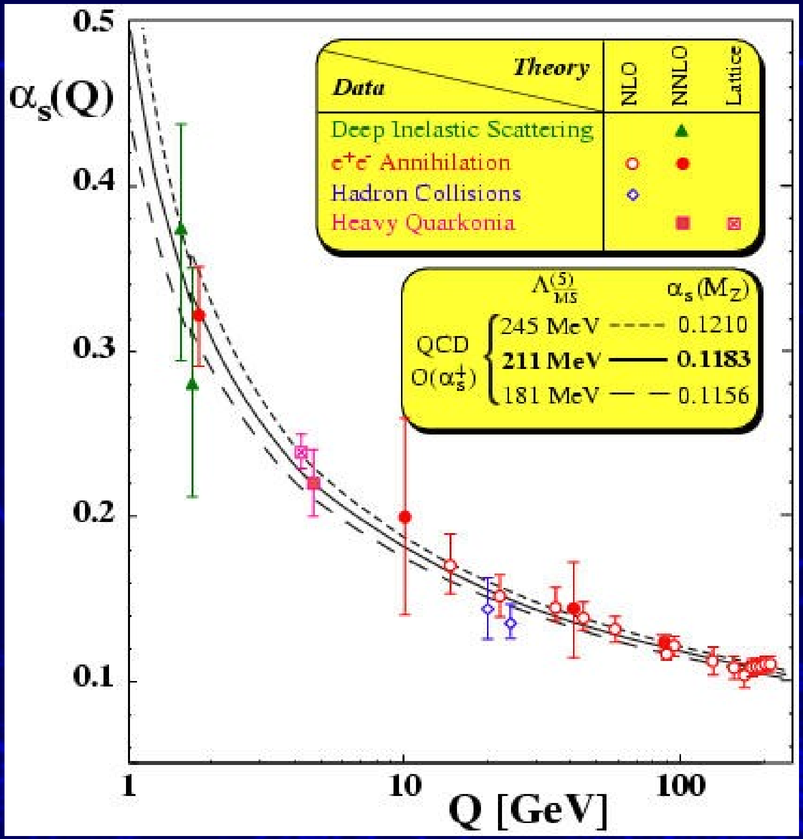
$$\alpha^{-1}(0) \approx 137 \leftrightarrow \alpha^{-1}(M_Z = 91.2\mathrm{GeV}) \approx 128.$$

- Note: Typically, scales are identified with energies

Strong interactions: QCD

Running coupling & asymptotic freedom in QCD:

- QCD: charge = colour ⇨ also gluons in the "cloud"
  - ⇨ charge screening becomes colour-antiscreening
  - ⇨ coupling stronger at large distance/small energies and weaker at small distances/ large energies
- This effect is called "asymptotic freedom".



Strong interactions: QCD

Running coupling & asymptotic freedom in QCD (cont'd)

- There is a connection between asymptotic freedom and "infrared slavery"
  ⇨ no confinement without this pattern of coupling
- Analytical form(s) of the strong coupling:

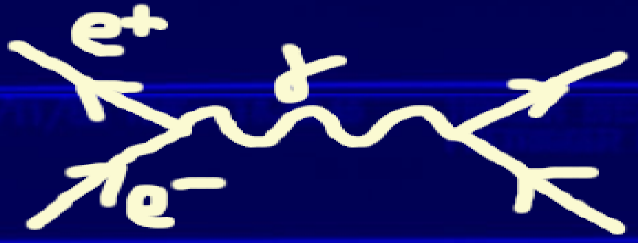
$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_S(\mu^2)\frac{33-2n_f}{12\pi} \cdot \ln\frac{Q^2}{\mu^2}} = \frac{1}{\frac{33-2n_f}{12\pi} \cdot \ln\frac{Q^2}{\Lambda^2}}$$

- Here, n_f = number of quarks with mass < Q and $\Lambda \sim 230$ MeV (the "Landau"-pole of QCD)

Strong interactions: QCD

Production of hadrons in e^+e^- collisions

- Basic diagram:
  at low energies only
  intermediate photon (no Z)



- Rate ~ | Diagram $|^2$ ~ e_f^2 for different fermions.

(e_f is fractional charge, 1 for muon, 2/3 for up, -1/3 for down)
⇨ Ratio of hadron (=quark) rate with respect to muon rate gives idea about the mass of the quark

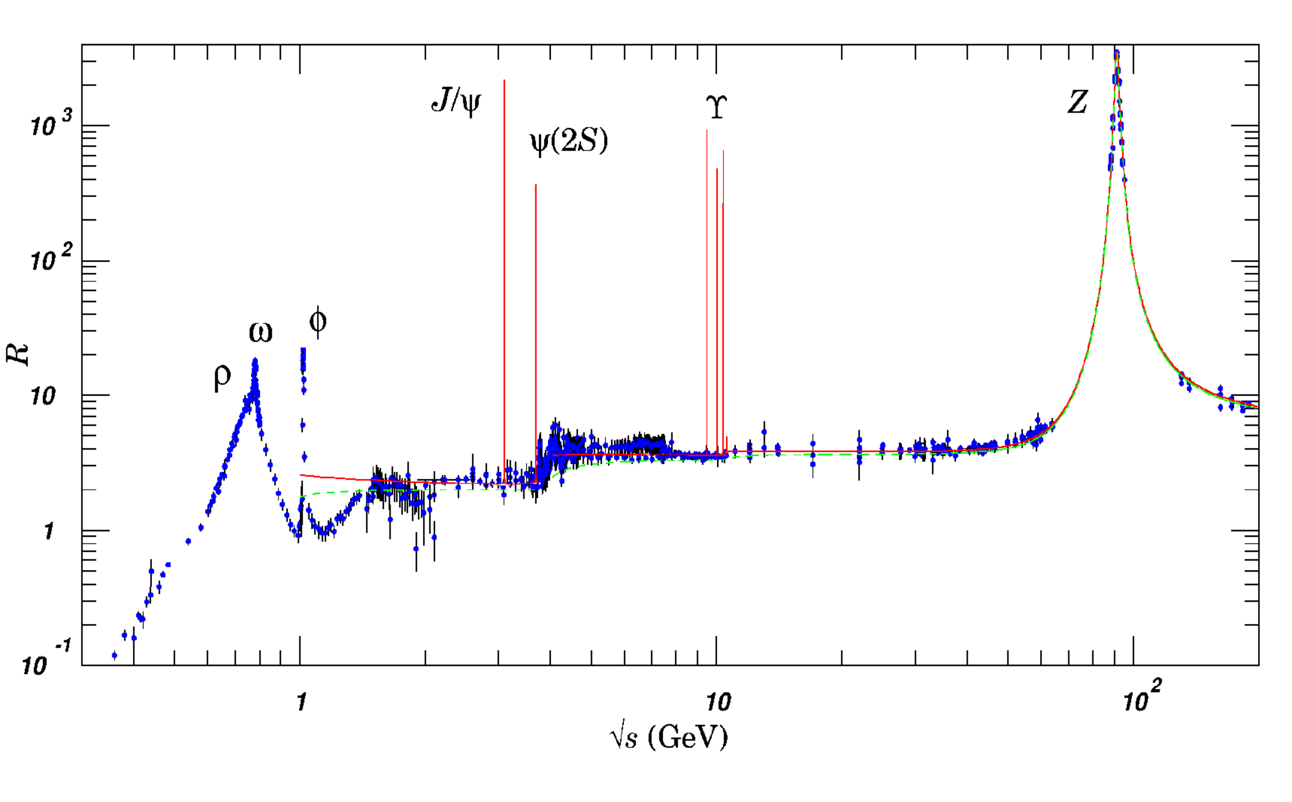
- Note: For the rates, keep in mind that each quark can come in three colours ⇨ multiply expression for rate with 3 when quarks are produced.

Strong interactions: QCD

Production of hadrons in e^+e^- collisions

- Ratio $R = 3\sum_{i\in q} e_i^2$

- "Spikes" due to resonances
- "Steps" due to opening of new channels.



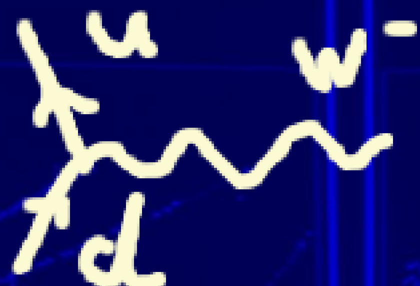
Weak interactions

Basic building blocks (coupling to fermions)

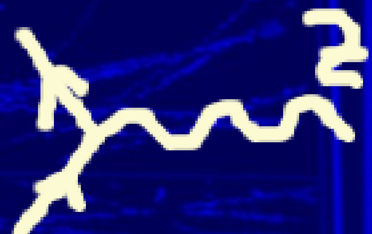
Weinberg-angle

$$\sim \frac{g_2}{\sqrt{2}} = \frac{e}{\sqrt{2}\sin\theta_W}$$

CKM-matrix



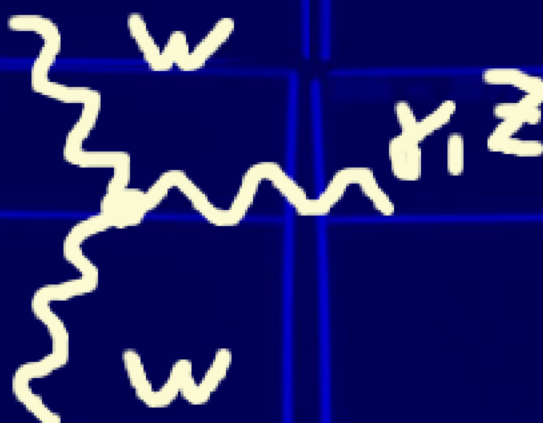
$$\sim \frac{g_2}{\sqrt{2}} \cdot V_{ud} = \frac{e}{\sqrt{2}\sin\theta_W} \cdot V_{ud}$$



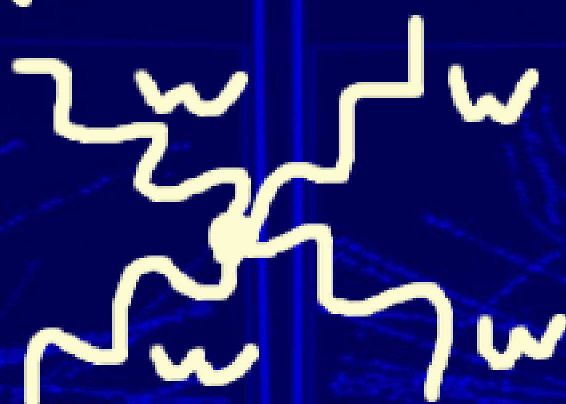
$$\sim \frac{g_2}{\cos\theta_W} \times \text{more complicated structure}$$

Weak interactions

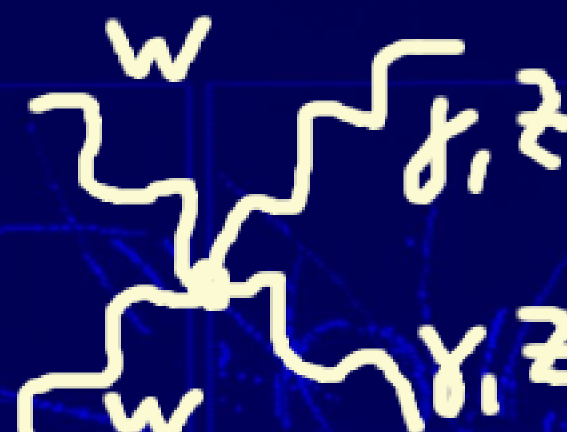
Basic building blocks (self-interactions)



$\sim e, g_W \cos\theta_W$



$\sim g_W^2$



$\sim e^2, e g_W \cos\theta_W, g_W^2 \cos^2\theta_W$

Weak interactions

Some examples:

- Muon/Tau decay into electron:

- Kaon-decay into pions:

Weak interactions

Parameters:

- Couplings: electromagnetic coupling $e = \sqrt{4\pi\alpha}$
- Weak coupling: $g_W = \frac{e}{\sin\theta_W}$, where $\sin^2\theta_W \approx 0.23$
- Cabibbo-Kobayashi-Maskawa matrix (CKM-matrix)

$$V_{qq'} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

here q = (u,c,t) and q' = (d,s,b). $\lambda \approx 0.22$, $A \approx 0.8$

- Note: Complex matrix triggers CP violation.

Weak interactions

CKM elements (cont'd):

- Efficient representation: triangles – multiply rows and columns, must yield 0 or 1 – CKM matrix is unitary (probability conservation)!

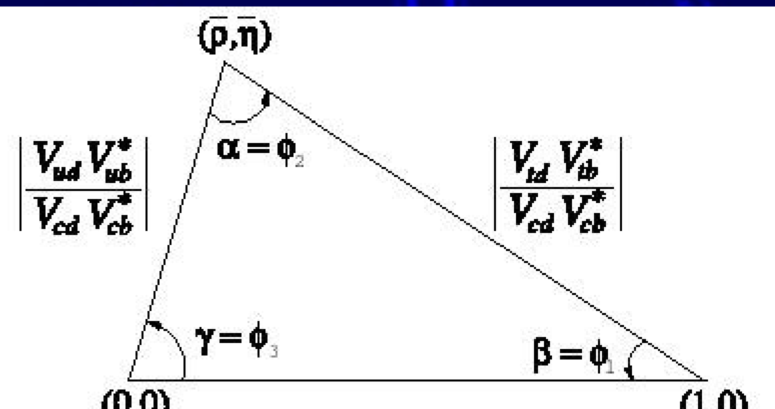
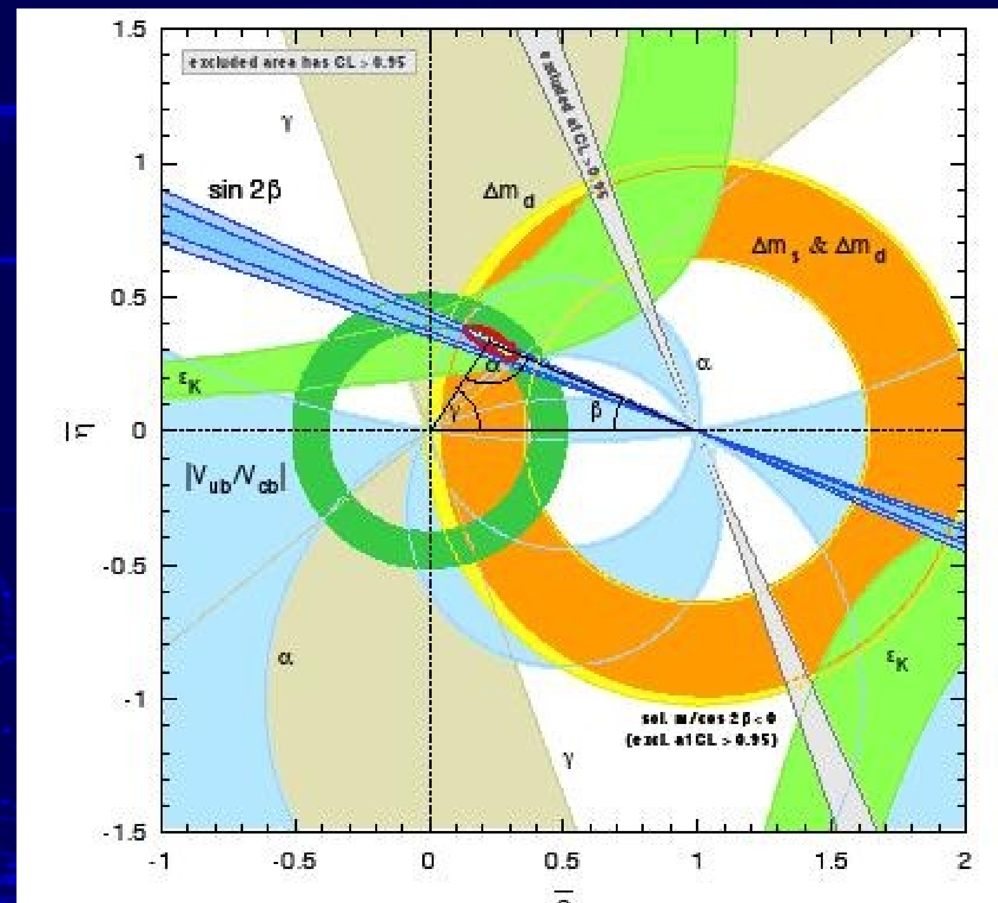


Figure 11.1: Sketch of the unitarity triangle.



The Higgs boson

- Problem so far:
  Theory ➪ all particles must be massless.
  (at odds with experimental facts)

- Reason (for completeness):
  - Theory formulated in terms of a Lagrangian with terms relevant for interactions, masses, kinetic energies etc..
  - Lagrangian must respect symmetries (in particular, gauge symmetries). Simple idea: Lagrangian must be invariant under field transformations

$$\phi(x) \to \phi'(x) = \exp[-i\theta(x)]\phi(x)$$

  - This cannot be guaranteed for mass-terms of the kind

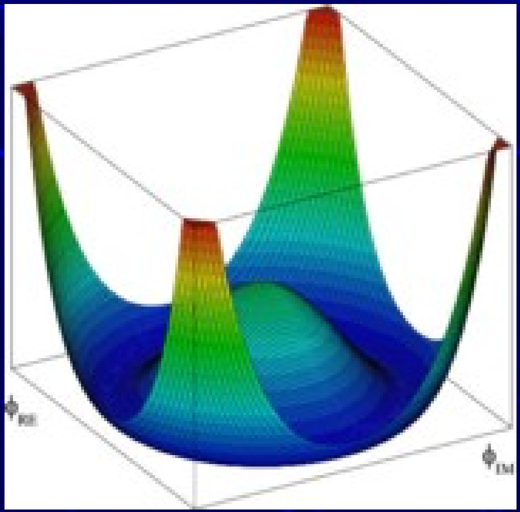
$$\mathcal{L}_{\rm mass} = m^2 W^\mu W_\mu \ldots$$

for real fields.

The Higgs boson

- Solution: A trick.
- Introduce a new set of (scalar, spin-0) fields.
- Give them a potential of the form

  $\mathcal{V} = -\mu^2|\Phi|^2 + \lambda|\Phi|^4$



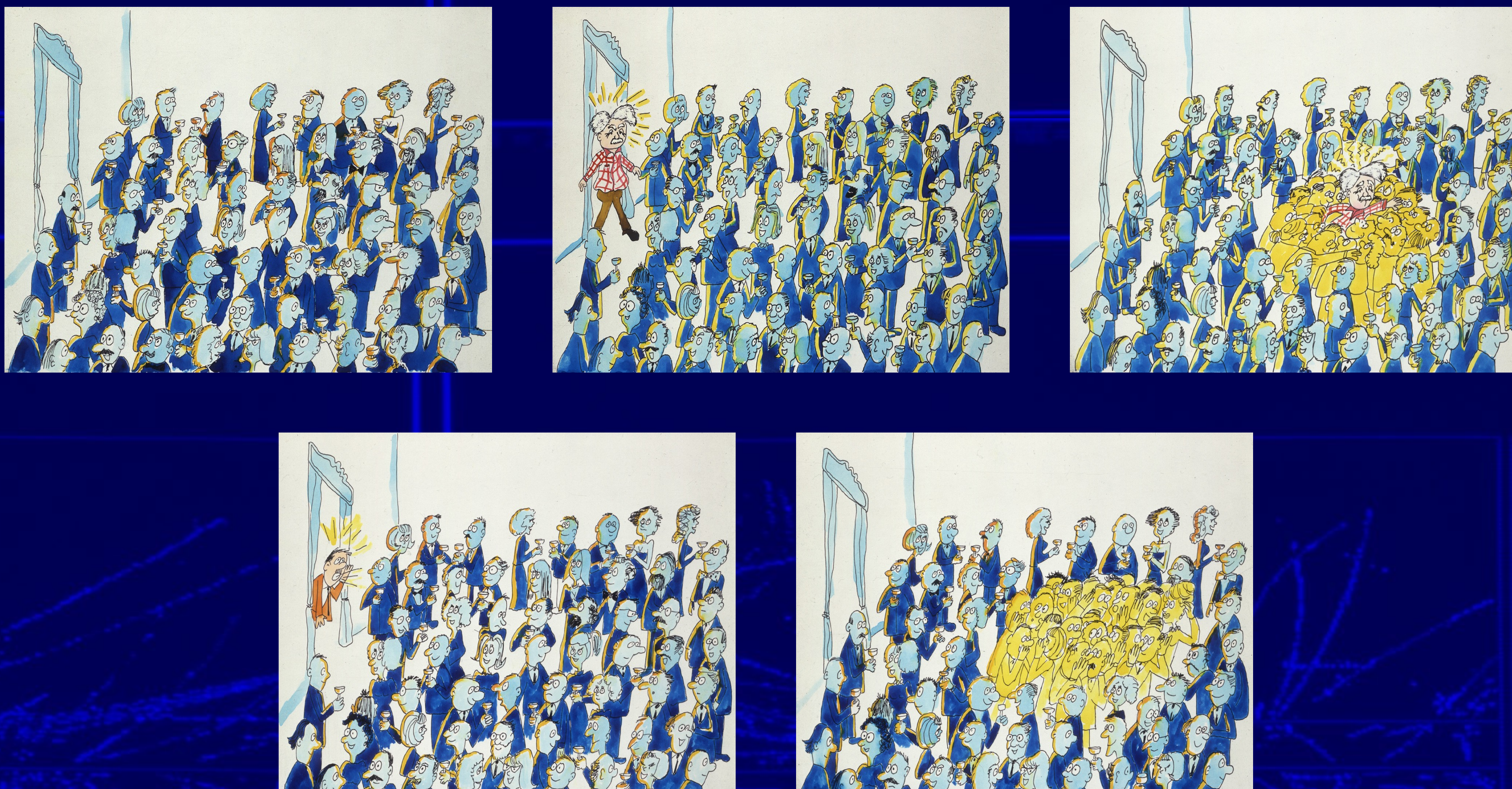
- yields non-trivial minima at

  $|\Phi|^2 = \mu^2/\lambda$

- gives "vacuum expectation value" $v = \sqrt{\mu^2/\lambda}$, acts like a "viscous medium" on the other fields (particles) ⇨ one of the scalar fields "survives" ⇨ the Higgs boson, still not found !

The Higgs boson

Particle masses and the Higgs boson, pictorially



The Higgs boson

Particle degrees of freedom:

- Particles have internal degrees of freedom:
  - Spin-1/2 fermions have 2 (spin-) degrees of freedom:
    spin up and down,
    measurable in inhomogeneous magnetic fields
    (the Stern-Gerlach experiment)
  - Massless spin-1 bosons (photons, gluons) have 2 polarisation degrees of freedom:
    e.g. left- and right-circular
  - Massive spin-1 bosons (W, Z) have 3 polarisation degrees of freedom (by the Higgs/Goldstone trick, some of the scalar fields act as the third polarisation)

The Higgs boson

Feynman rules for the surviving Higgs boson:

- Note: It has spin-0 and charge 0.
- Some interactions (there are also 4-vertices):

$$\sim \frac{m_f}{v} \qquad \sim \frac{M_V}{v} \qquad \sim \frac{M_H}{v}$$

- The vacuum expectation value v can be inferred from the masses of the W and Z bosons:

$$v \approx 246 \mathrm{GeV}.$$

Conservation laws

How conservation laws are realised

- Each vertex respects all symmetries (conservation laws) of the full theory:
  - Strict energy and momentum conservation
    (this is different from time-dependent perturbation theory!)
  - Conservation of charges (including colour)
  - Conservation of lepton and baryon number
    (no vertices with two different kind of leptons or one lepton and/or quark only!)
  - No conservation of flavour in charged weak interactions (with W's), parametrised by CKM matrix ⇨ important for decays.