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Correlated adiabatic and isocurvature CMB fluctuations after the WMAP

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Based on **astro-ph/0304175**
by J. Väliviita and V. Muhonen,
[Phys. Rev. Lett. 91, 131302 (2003)].

Outline

- Adiabatic and CDM isocurvature initial fluctuations
- Correlation between them?
- The resulting angular power spectra
- Constraints from the WMAP data
- Large optical depth τ or correlated initial conditions?

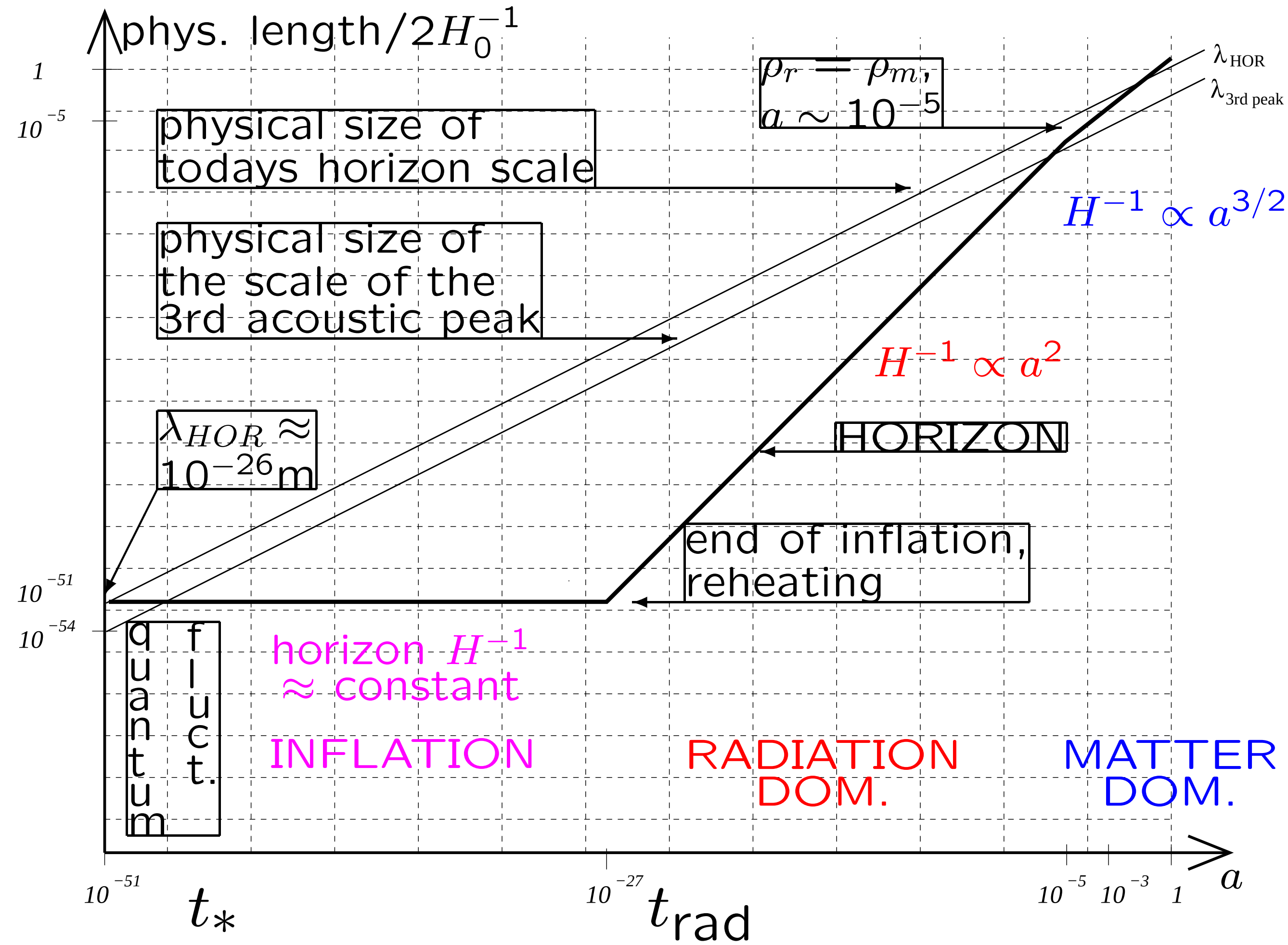
(Explanation for slide 1)

I'll talk about a possible correlation between adiabatic and cold dark matter (CDM) isocurvature initial cosmic microwave background (CMB) fluctuations in the wake of the Wilkinson Microwave Anisotropy Probe (WMAP) data.

First I just remind you about the difference of the adiabatic and isocurvature initial conditions by showing two naive pictures. Then I mention some earlier results for uncorrelated initial fluctuations and after that I'll go to the correlation stuff.

Prior to the WMAP only some specific correlated models (with many parameters fixed, see [T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001), Phys. Rev. D **66**, 063501 (2002)]) could be constrained, but it wasn't possible to make reasonable constraints on the *general* correlated models. Now the accurate enough TT (temperature-temperature, i.e. temperature auto correlation C_l^{TT}) and TE (temperature-polarization E-mode cross-correlation C_l^{TE}) power spectra are available. The last two points in my talk are based on the Physical Review Letters article by me and Vesa Muhonen [J. Valiviita and V. Muhonen, Phys. Rev. Lett. **91**, 131302 (2003)]. Unfortunately (or should I say fortunately) some of you have heard quite similar talk by me in Blois in France [J. Valiviita, astro-ph/0310206].

History of fluctuations (Slide 2)



(Explanation for slide 2)

Let's start by some history of the CMB fluctuations. The horizontal axis is the scale factor of the universe (so that time grows towards right) and the vertical axis is the physical length scale. As the universe expands the physical lengths get stretched also.

First there is an inflationary period, end of inflation and reheating after which the universe is radiation dominated. Much later the matter starts to dominate. The bold solid black line is the inverse of the Hubble parameter (H^{-1}), which is roughly the horizon length. It is more or less constant during inflation and then starts to grow.

During inflation there are quantum fluctuations that freeze in when a particular scale goes out of the horizon. Here I show two different scales by thin solid black lines: today's horizon scale and the scale corresponding to the third acoustic peak.

This slide is to explain two instants of time appearing on my later slides. The first instant is the horizon exit of cosmologically interesting scales during inflation. I mark it by t_*. However, the initial conditions for CMB angular power calculations (like CAMB code) should be given deep in the radiation dominated era. Thus the interesting "initial time" for CMB physicists is $t_{\rm rad}$.

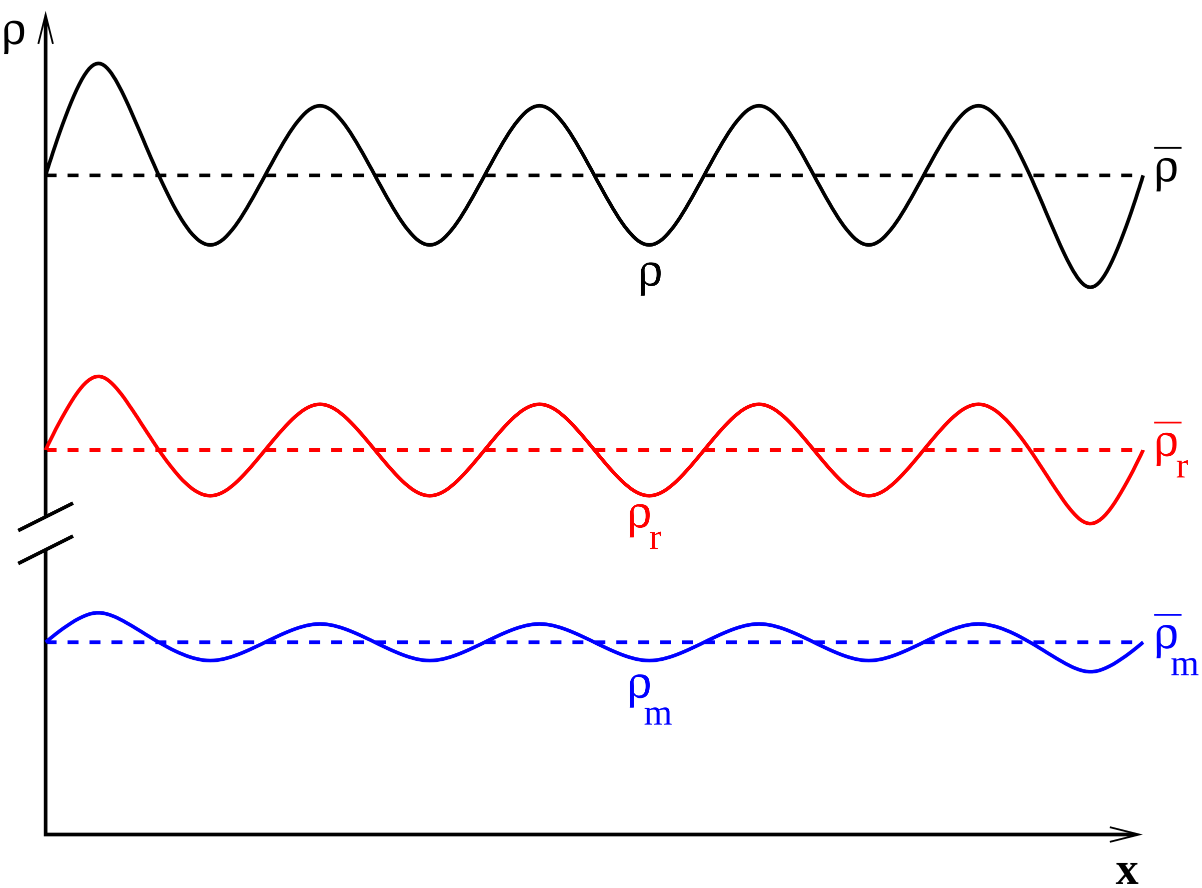
Adiabatic initial fluctuations (Slide 3)

For the adiabatic fluctuations there is no entropy perturbation

$$S_{\mathrm{rad}} \equiv S_{c\gamma} = \frac{\delta(n_c/n_\gamma)}{n_c/n_\gamma} = \frac{\delta\rho_c}{\bar{\rho}_c} - \frac{3}{4}\frac{\delta\rho_\gamma}{\bar{\rho}_\gamma} \equiv 0\,,$$

but the total energy density fluctuates or, more precisely, there is perturbation in the comoving curvature

$$\langle|\mathcal{R}|^2_{\mathrm{rad}}\rangle \neq 0.$$



(Explanation for slide 3)

Then about the initial conditions at the beginning of the radiation dominated era.

The most studied possibility is pure adiabatic case. Then there is no entropy perturbation at t_{rad}, but the total energy density fluctuates. In the figure I have an example. The spatial fluctuation in matter and radiation energy density is in the same phase so that they yield to an initial fluctuation in the total energy density.

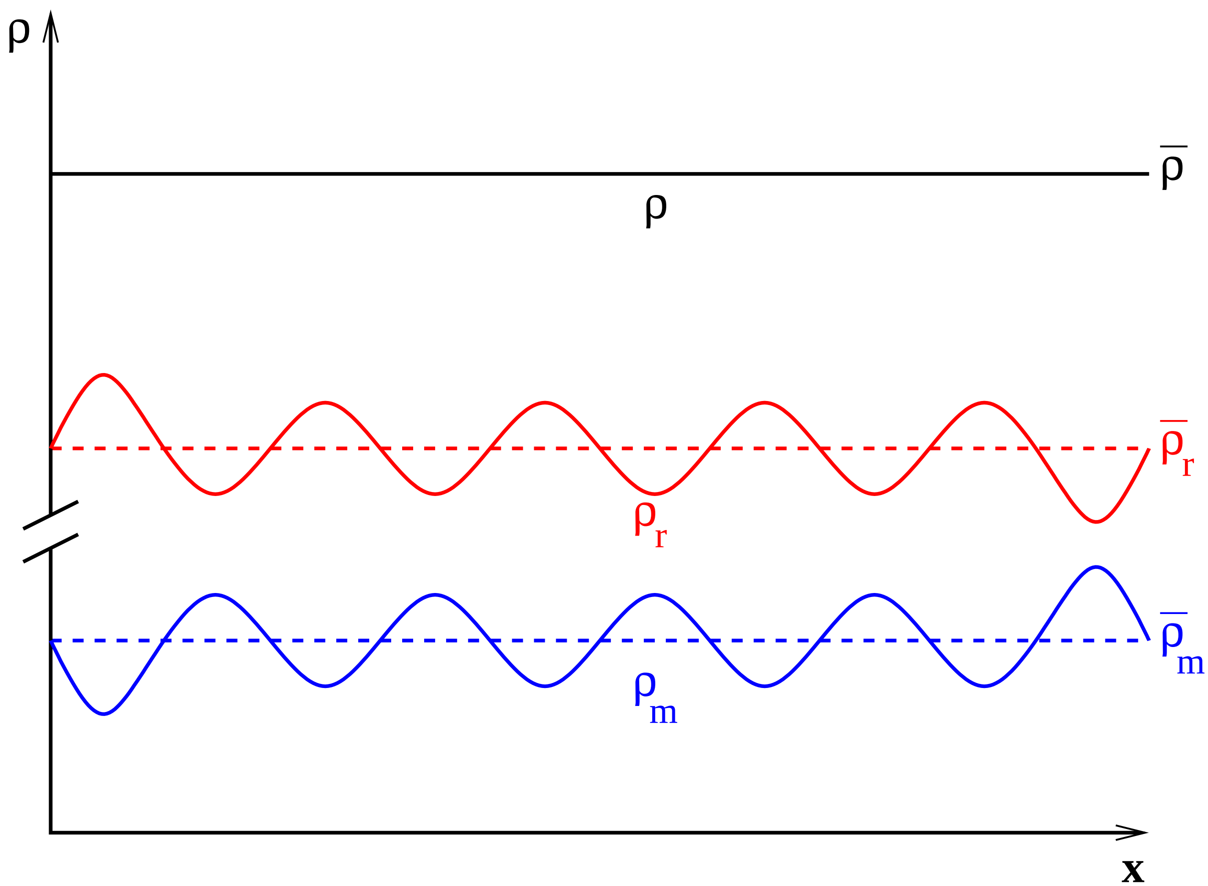
Isocurvature initial fluctuations (Slide 4)

Now the entropy fluctuates initially (at the beginning of radiation dominated era)

$$S_{\mathrm{rad}} \equiv S_{c\gamma} = \frac{\delta\rho_c}{\bar{\rho}_c} - \frac{3}{4}\frac{\delta\rho_\gamma}{\bar{\rho}_\gamma} \neq 0,$$

but there is no perturbation in the total energy density,

$$\delta\rho = 0 \Rightarrow \delta\rho_\gamma = -\delta\rho_c\,, \quad \text{or} \quad \left\langle |\mathcal{R}|^2_{\mathrm{rad}} \right\rangle \approx 0\,.$$



(Explanation for slide 4)

In the isocurvature case the specific entropy fluctuates spatially but there is no initial fluctuations in the total energy density or more precisely in the comoving curvature $\mathcal{R}$. For example, the fluctuation in matter and radiation could cancel each other giving spatially constant total energy density.

(Slide 5)

Pure adiabatic, pure isocurvature, or mixture...

- In general case, the initial fluctuations are mixture of adiabatic and isocurvature perturbations.

- Already, e.g., Boomerang and Maxima data could be used to set strict 2σ constraints for uncorrelated mixture of adiabatic and CDM isocurvature initial perturbations in a flat universe ($\Omega_{\rm tot} = 1$) [K. Enqvist, H. Kurki-Suonio and J. Valiviita, Phys. Rev. D **62**, 103003 (2000)]
  - At most **56%** of the final temperature anisotropy angular power at the quadrupole ($l = 2$) could come from the isocurvature initial perturbations
  - At most **13%** of the power at the first acoustic peak ($l \approx 200$) could come from the isocurvature initial perturbations

- Pure CDM isocurvature fluctuations have been ruled out even in open or closed universe [K. Enqvist, H. Kurki-Suonio and J. Valiviita, Phys. Rev. D **65**, 043002 (2002)]

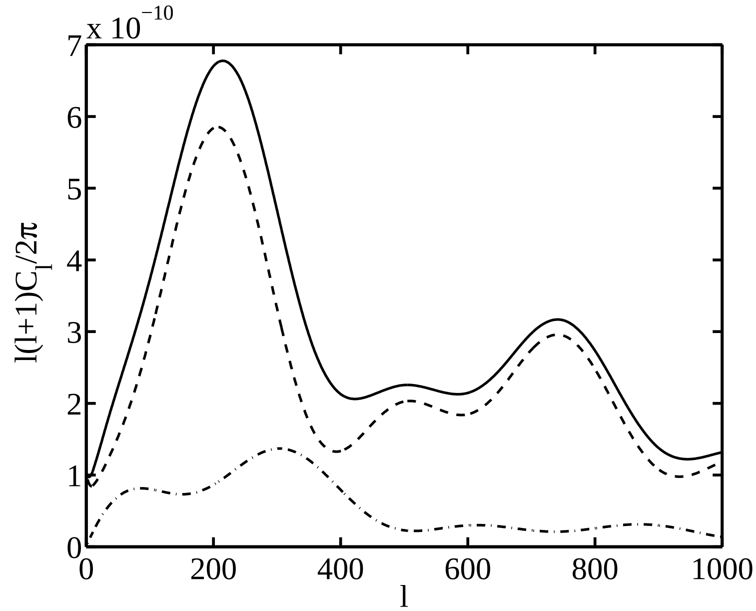
(Explanation for slide 5)

The evolution of perturbations is basically described by second order differential equations, adiabatic and isocurvature initial conditions being two independent modes. Thus the most general initial condition is a mixture of adiabatic and isocurvature fluctuations. The evolution equation tells how adiabatic and isocurvature initial fluctuations are converted into the temperature fluctuation present at the last scattering surface and finally into the presently observable temperature (or polarization) anisotropy described by the angular power spectrum that contains a series of peaks and valleys. If the other cosmological parameters are kept fixed, but one changes form adiabatic initial conditions to the isocurvature ones, then the resulting angular power spectra are roughly in the opposite phases.

Already the first data sets by Boomerang and Maxima could be used to constrain uncorrelated mixture of adiabatic and isocurvature fluctuations in flat universe models. We found the maximum 2σ allowed isocurvature contribution to the quadrupole ($l = 2$) temperature anisotropy to be 56% and to the first acoustic peak ($l \sim 200$) about 13% [K. Enqvist, H. Kurki-Suonio and J. Valiviita, Phys. Rev. D **62**, 103003 (2000) astro-ph/0006429]. Since in open universe all the features of the angular power spectrum are shifted towards right (larger l, smaller scales) and in closed universe towards left, it still seemed to be possible to fit the first acoustic peak by open or closed pure isocurvature models. However, the second data releases by Boomerang and Maxima identified also the second acoustic peak so well that the "adiabatic peak structure" became evident and we could finally rule out all pure CDM isocurvature models [K. Enqvist, H. Kurki-Suonio and J. Valiviita, Phys. Rev. D **65**, 043002 (2002) astro-ph/0108422]. Nevertheless, *mixed* correlated or uncorrelated models remain as an interesting possibility.

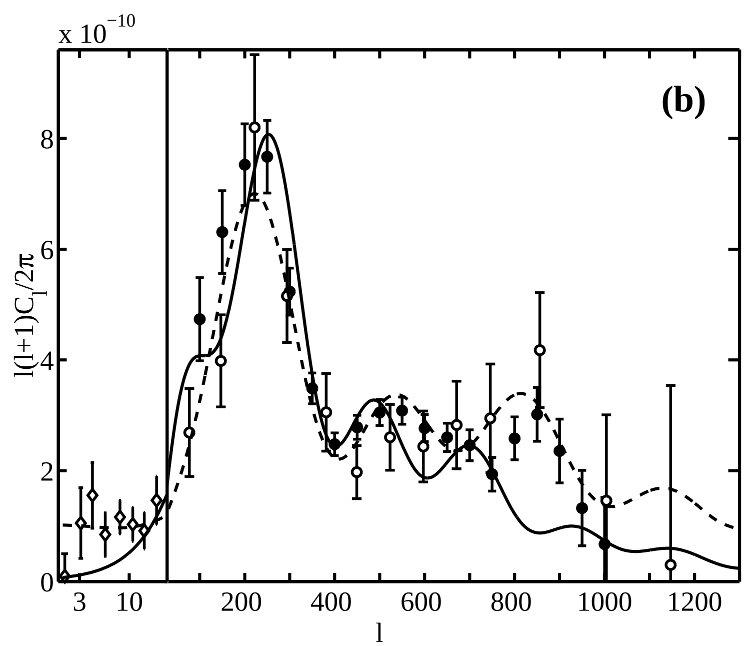
(Slide 6)

From K. Enqvist, H. Kurki-Suonio and J. Valiviita, Phys. Rev. D **62**, 103003 (2000)



Maximum 2σ allowed isocurvature contribution to $l = 200$.

From K. Enqvist, H. Kurki-Suonio and J. Valiviita, Phys. Rev. D **65**, 043002 (2002)



– – – typical adiabatic model, — best CDM isocurvature model

(Explanation for slide 6)

The upper panel: Here I show the angular power of an uncorrelated flat universe ($\Omega = 1$) model with maximum isocurvature contribution to the first acoustic peak. The solid line is the total angular power while the dashed line represents the adiabatic component and dot-dashed line the isocurvature component. The maximum isocurvature contribution to the first acoustic peak allowed by Boomernag, Maxima and COBE data is 13 %.

The lower panel: The solid line is the best-fit pure isocurvature model. This is a closed universe model with $\Omega = 1.09$. The first eight data points are from COBE and the remaining points from the second data releases by Boomerang and Maxima. Comparing the best-fit pure isocurvature model to the typical well-fitted pure adiabatic model one can even by an eye see that pure CDM isocurvature models are ruled out.

Correlation... (Slide 7)

- The first studies by [D. Langlois, Phys. Rev. D **59**, 123512 (1999)].
- Generally

$$\begin{pmatrix}\hat{\mathcal{R}}_{\text{rad}}(k)\\ \hat{\mathcal{S}}_{\text{rad}}(k)\end{pmatrix} = \begin{pmatrix}1 & T_{\mathcal{RS}}(k)\\ 0 & T_{\mathcal{SS}}(k)\end{pmatrix}\begin{pmatrix}\hat{\mathcal{R}}_*(k)\\ \hat{\mathcal{S}}_*(k)\end{pmatrix},$$

[L. Amendola, C. Gordon, D. Wands and M. Sasaki, Phys. Rev. Lett. **88**, 211302 (2002); C. Gordon, D. Wands, B. A. Bassett and R. Maartens, Phys. Rev. D **63**, 023506 (2001)].

- The initial curvature, $\hat{\mathcal{R}}_{\text{rad}}$, and entropy, $\hat{\mathcal{S}}_{\text{rad}}$, perturbations are usually approximated by power laws:

$$\hat{\mathcal{R}}_{\text{rad}} = A_r\left(\frac{k}{k_0}\right)^{n_1}\hat{a}_r(\mathbf{k}) + A_s\left(\frac{k}{k_0}\right)^{n_3}\hat{a}_s(\mathbf{k}),$$
$$\hat{\mathcal{S}}_{\text{rad}} = B\left(\frac{k}{k_0}\right)^{n_2}\hat{a}_s(\mathbf{k}),$$

[C. Gordon, astro-ph/0112523; H. V. Peiris *et al.* (WMAP group), astro-ph/0302225].

$\Rightarrow$ Correlation between adiabatic and isocurvature

$$\left\langle\hat{\mathcal{R}}(\mathbf{k})\hat{\mathcal{S}}^*(\mathbf{k}')\right\rangle\Big|_{\text{rad}} = A_sB\left(\frac{k}{k_0}\right)^{n_3+n_2}\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$
$$= A^2 f_{\text{iso}}\cos\Delta\left(\frac{k}{k_0}\right)^{n_3+n_2}\delta^{(3)}(\mathbf{k}-\mathbf{k}'),$$

where $A^2 = A_r^2 + A_s^2$, $f_{\text{iso}} = |B/A|$ and $\cos\Delta = \text{sign}(B)A_s/A$.

- WMAP team assumed $n_1 = n_3$.

(Explanation for slide 7)

As I told earlier, the initial fluctuations for the CMB physicist are the comoving curvature perturbation $\widehat{\mathcal{R}}_{\text{rad}}(k)$ and the entropy perturbation $\widehat{\mathcal{S}}_{\text{rad}}(k)$ at the beginning of the radiation dominated era. These are related to the perturbations at the horizon exit during inflation by the given formula, where $T_{\mathcal{RS}}(k)$ and $T_{\mathcal{SS}}(k)$ are the transfer functions that describe the evolution of the perturbations form the time of the horizon exit to the beginning of the radiation dominated era. In most cases, they are found only numerically by solving the evolution equations and modeling the reheating process.

The initial perturbations $\widehat{\mathcal{R}}_{\text{rad}}(k)$ and $\widehat{\mathcal{S}}_{\text{rad}}(k)$ are usually approximated by power laws, which is a good approximation assuming that "everything" changes slowly during inflation. Actually, e.g., in some cases of double inflation this is not true.

In an ordinary pure adiabatic case one would have only the first term $A_r(k/k_0)^{n_1}\widehat{a}_r(\mathbf{k})$. Now we have additional terms coming from the entropy perturbation during inflation. In my terminology $A_r(k/k_0)^{n_1}\widehat{a}_r(\mathbf{k})$ is called the first adiabatic component, $A_s(k/k_0)^{n_3}\widehat{a}_s(\mathbf{k})$ the second adiabatic component (generated by the entropy perturbation during inflation if the trajectory in the multi field space is curved), and $B(k/k_0)^{n_2}\widehat{a}_s(\mathbf{k})$ the isocurvature component. A_r, A_s, and B are amplitudes and n_is spectral indices. Moreover, $\widehat{a}_r$ and $\widehat{a}_s$ are Gaussian random variables obeying

$$\langle \widehat{a}_r \rangle = 0, \quad \langle \widehat{a}_s \rangle = 0, \quad \langle \widehat{a}_r(\mathbf{k})\widehat{a}_s^*(\mathbf{k}')\rangle = \delta_{rs}\delta^{(3)}(\mathbf{k}-\mathbf{k}')\,.$$

Now there is a 100 % correlation between the second abiabatic and the isocurvature component. This correlation can be parametrized by some overall amplitude A, the isocurvature fraction f_{iso}, and the relative correlation amplitude $\cos\Delta$.

WMAP team already analyzed this kind of models, but for simplicity, they assumed the two adiabatic spectral indices to be equal, $n_1 = n_3$. This is not well motivated theoretically, since n_1 comes from the curvature perturbation and n_3 from the entropy perturbation. If one really needs to simplify the analysis, one should consider to put n_3 and n_2 equal.

The final angular power and spectral indices

- The angular power spectrum is now given by

$$\begin{aligned} C_l = A^2\Big[\sin^2(\Delta)C_l^{\rm ad1} &+ \cos^2(\Delta)C_l^{\rm ad2} \\ +f_{\rm iso}^2 C_l^{\rm iso} &+ f_{\rm iso}\cos(\Delta)C_l^{\rm cor}\Big]. \end{aligned}$$

- For convenience, define new spectral indices as

$$\begin{aligned} n_{\rm ad1} - 1 &= 2n_1 \\ n_{\rm ad2} - 1 &= 2n_3 \\ n_{\rm iso} - 1 &= 2n_2 \\ \Big[n_{\rm cor} &= (n_{\rm ad2} + n_{\rm iso})/2\Big]. \end{aligned}$$

- We used a grid method to scan the parameter space
  - τ, Ω_Λ, ω_b, ω_c, $n_{\rm ad1}$, $n_{\rm ad2}$, $n_{\rm iso}$, $f_{\rm iso}$, and $\cos(\Delta)$

    $\sim 10^{10}$ combinations
  - C_l's calculated by our modified version of CAMB.
  - Some likelihoods are non-Gaussian $\Rightarrow$ marginalization by **integration** is adopted.

(Explanation for slide 8)

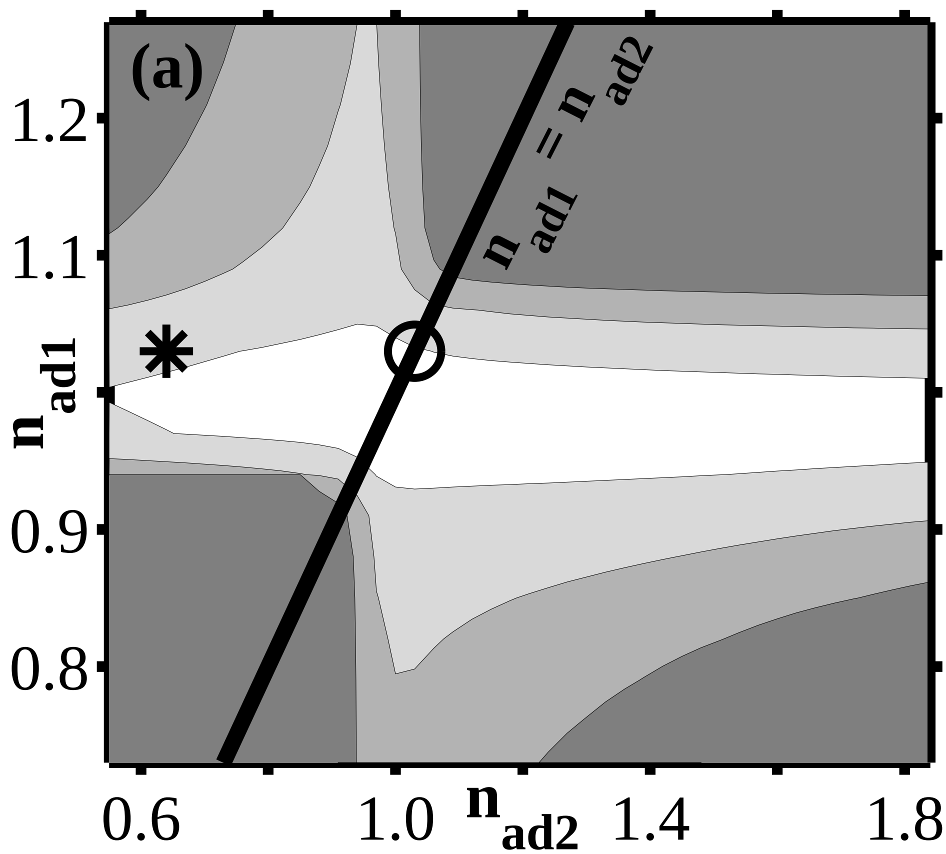
Now the final angular power spectrum will be a combination of four different components: the first adiabatic, the second adiabatic, the isocurvature and the correlation between the second adiabatic and the isocurvature component.

Historically the spectral indices have been defined so that $n = 1$ stands for the scale-free spectrum. Thus we need to redefine our spectral indices to match this convention.

Our results

(Slide 9)

- The data do allow, but do not especially favour models with equal adiabatic spectral indices.



$*$ = our best-fit model

$\circ$ = the best-fit $n_{\rm ad2} = n_{\rm ad1}$ model

- Actually, the data favour models where the two adiabatic components have opposite spectral tilts ($n_{\rm ad1} > 1$ and $n_{\rm ad2} < 1$ or vice versa).

(Explanation for slide 9)

We found that the data do not especially favour the WMAP team restriction. They constrained their analysis on the solid line $n_{\mathrm{ad2}} = n_{\mathrm{ad1}}$ only. As you can see, the data favour (or at least allows) the regions where the two adiabatic components have opposite spectral tilts.

The colour codes in the figure are: The 68.3%/1σ (white), 95.4%/2σ (light gray), 99.7%/3σ (medium gray), and more than 3σ (dark gray). This means that the data favour white and light gray regions.

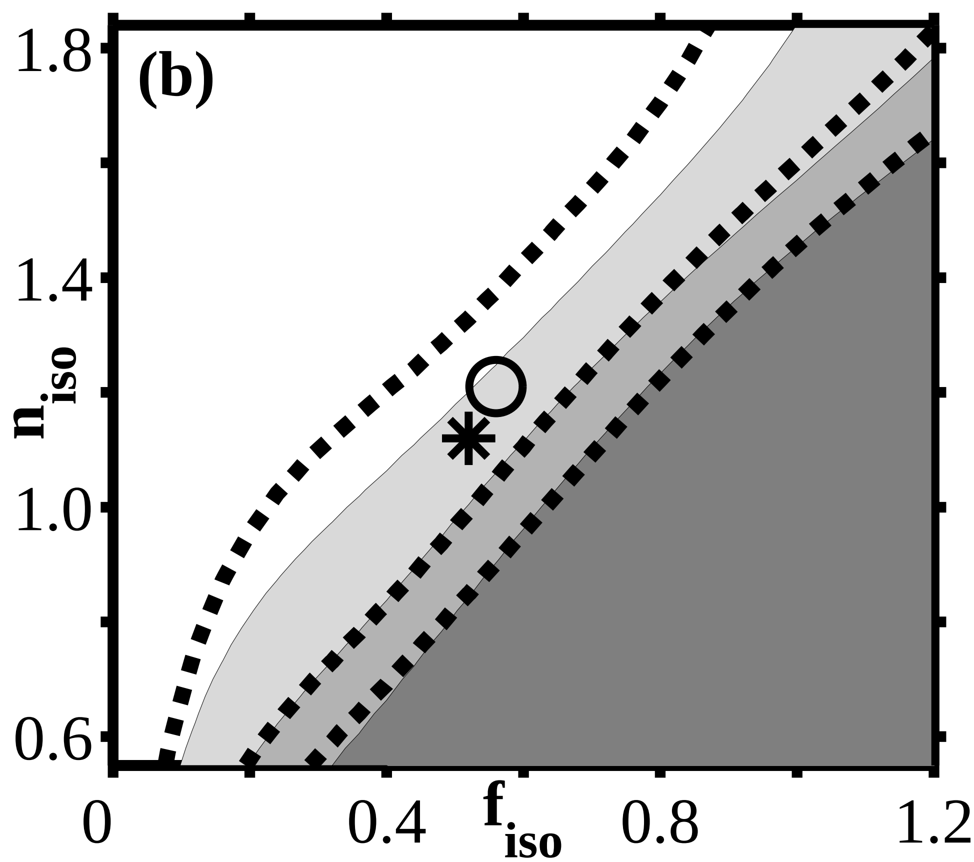
(Slide 10)

- The 2σ upper bound for the isocurvature fraction in the initial power is

$$f_{\rm iso} < 0.84$$

using a prior $n_{\rm iso} < 1.84$.

- The same calculation with WMAP team restriction ($n_{\rm ad2} = n_{\rm ad1}$) would give a slightly stricter upper bound, $f_{\rm iso} < 0.74$.



(Explanation for slide 10)

Using the WMAP data only we got for the 2σ upper bound of isocurvature fraction $f_{\text{iso}} < 0.84$. Since the angular power alone cannot give "any" upper bound for the isocurvature spectral index, we had to assume some prior for that $n_{\text{iso}} < 1.84$. If we had chosen a larger prior, the upper bound for the isocurvature fraction would have been larger also as is evident from the figure. However, we assume from the WMAP team analysis that the large scale structure data (2dF) would give about $n_{\text{iso}} < 1.80$ motivating our prior. In any case, we are able to compare our result to the result which we would get with the WMAP team restriction $n_{\text{ad2}} = n_{\text{ad1}}$. With this simplification one would get unrealistically strict upper bound for f_{iso}.

Colour codes are same as on slide 9. In addition dashed lines present confidence levels for $n_{\text{ad2}} = n_{\text{ad1}}$ models for comparison. 1σ region differs significantly from our general models but 2σ regions are nearly identical.

Running (adiabatic) spectral index? (Slide 11)

- The adiabatic initial power is

$$\mathcal{P}_{\mathcal{R}} = A^2 \left[\sin^2\Delta (\tfrac{k}{k_0})^{n_{\text{ad1}}-1} + \cos^2\Delta (\tfrac{k}{k_0})^{n_{\text{ad2}}-1}\right] .$$

- If n_{ad1} and n_{ad2} are nearly equal or $\Delta = 0$, $\pi/2$, or π, then the adiabatic power can well be approximated by a single power law

$$\mathcal{P}_{\mathcal{R}} = D(\tfrac{k}{k_0})^{n_{\text{ad}}-1} ,$$

where

$$n_{\text{ad}} - 1 = \frac{d \ln \mathcal{P}_{\mathcal{R}}(k)}{d \ln k}.$$

- The first derivative of this is always non-negative

$$\frac{dn_{\text{ad}}(k)}{d\ln k} = \frac{\sin^2\Delta\cos^2\Delta(n_{\text{ad1}} - n_{\text{ad2}})^2 k^{n_{\text{ad1}}+n_{\text{ad2}}}}{[\sin^2\Delta k^{n_{\text{ad1}}} + \cos^2\Delta k^{n_{\text{ad2}}}]^2}.$$

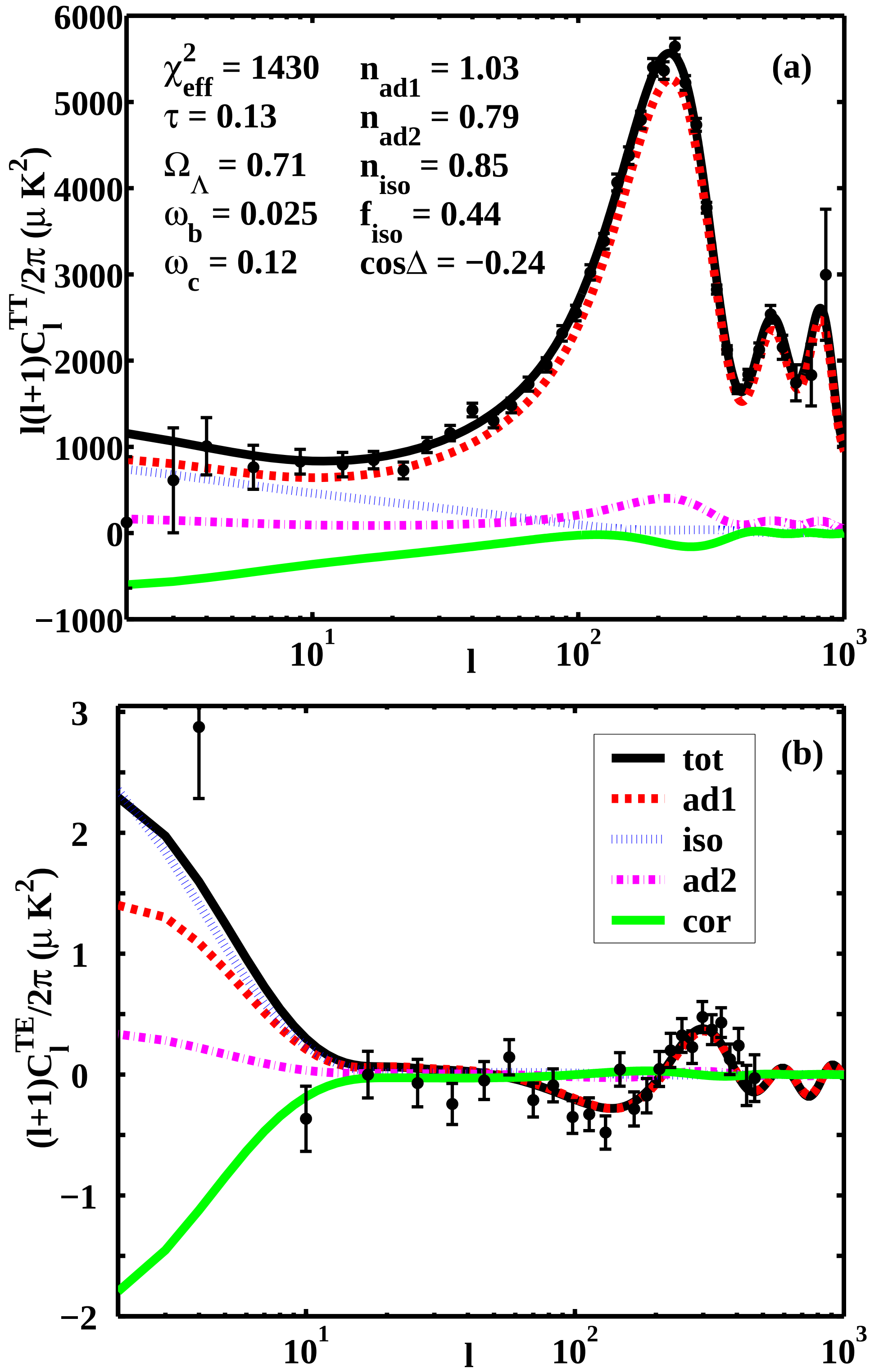
- The WMAP group observed that the combined CMB and other cosmological data may favor a running spectral index with a *negative* first derivative.

(Explanation for slide 11)

What I have told so far may naturally lead to a running adiabatic spectral index. So recall that the adiabatic initial power is a sum of two components. If the spectral indices are equal, we can write the adiabatic initial power as a single power law. Same happens if the sine or cosine is zero. In all other cases an attempt to write the adiabatic part in terms of a single power law leads to a scale dependent spectral index with the first derivative given by the last formula of the slide. This is always non-negative.

WMAP team reported that combining their data with other data sets may favour a running spectral index with a negative first derivative. Thus one could expect that the combined data tend to force the two adiabatic spectral indices to be nearly equal. However, the correlation power may change the situation dramatically and that's why we are doing a more sophisticated analysis on this issue.

Example 2σ allowed angular power (Slide 12)



(Explanation for slide 12)

Here we have an example of a two sigma allowed correlated model. The upper panel gives the TT power and the lower panel the TE power. The total angular power (solid black line) is a sum of four components: the first adiabatic component (dashed red), the second adiabatic component (dot-dashed magenta), the isocurvature component (dotted blue), and the correlation component (solid green).

In negatively correlated models, as here, the isocurvature contribution to the lowest multipoles l in TT spectrum can be quite large due to cancellation by the correlation part. In TE spectrum the cancellation is not as exact so that there one gets enough power at the lowest multipoles. Actually, in this particular example, the isocurvature component dominates the TE spectrum at the quadrupole $l = 2$. That's why even quite a small optical depth due to reionization ($\tau = 0.13$) is enough to give a reasonable TE power at the quadrupole. Thus in correlated models the measured high quadrupole in TE spectrum can be achieved without having as large τ as reported by the WMAP team. I mean that one should keep in mind a possible degeneration between optical depth and correlation/isocurvature contribution.

On the other hand, positive correlation together with isocurvature could nicely lead to even more power in TE at the quadrupole, but unfortunately it tends to give a high quadrupole also in TT, which is not favoured by data. An interesting possibility to explain the low quadrupole in TT would be to have a very strong negative correlation which "eats" some power from the TT quadrupole. Then in the TE this could be compensated by a large τ. Suitably strong correlation contribution to the total TT power requires that the second adiabatic and isocurvature component dominate over the first adiabatic component.

Conclusions (Slide 13)

- Correlation is certainly allowed by the data.

- The WMAP data tend to favour models where $n_{\text{ad1}} \neq n_{\text{ad2}}$.

  $\Rightarrow$ Running effective adiabatic spectral index.

- $f_{\text{iso}} < 0.84$ (when using the WMAP data only and a prior $n_{\text{iso}} < 1.84$).

- We are making a more refined analysis on the WMAP and other data to have more quantitative results, e.g., reliable upper bound for isocurvature fraction.

- Correlation could have some effect to the optical depth due to reionization (τ) interpreted from the measured data.

  - When allowing for a correlated mixture of adiabatic and isocurvature fluctuations, even a smaller τ could possibly give a reasonable fit to the (quadrupole in the) TE power spectrum!

References

This talk is based on: J. Valiviita and V. Muhonen, "Correlated adiabatic and isocurvature CMB fluctuations in the wake of WMAP," Phys. Rev. Lett. **91**, 131302 (2003) [astro-ph/0304175].

I gave nearly similar talk in Blois in France. See my contribution to Blois proceedings: J. Valiviita, "Correlated adiabatic and isocurvature CMB fluctuations in the light of the WMAP data," astro-ph/0310206.

Special correlated models: T. Moroi and T. Takahashi, Phys. Lett. B **522**, 215 (2001) [Erratum-ibid. B **539**, 303 (2002)] [hep-ph/0110096]; Phys. Rev. D **66**, 063501 (2002) [hep-ph/0206026].

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