UNCERTAINTIES AND DISCOVERY POTENTIAL IN PLANNED EXPERIMENTS

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Abstract
We describe a method for estimation of the discovery potential on new physics in planned experiments. Application of the test of equal-probability allows to estimate the exclusion limits on new physics. We also estimate the influence of systematic uncertainty related to nonexact knowledge of signal and background cross sections on the discovery probability of new physics in planned experiments. An account of such systematics is very important in the search for supersymmetry at LHC.

1 INTRODUCTION

One of the common goals in the forthcoming experiments is the search for new phenomena. In estimation of the discovery potential of the planned experiments the background cross section (for example, the Standard Model cross section) is calculated and, for the given integrated luminosity L, the average number of background events is $n_b = \sigma_b \cdot L$. Suppose the existence of a new physics leads to additional nonzero signal cross section σ_s with the same signature as for the background cross section that results in the prediction of the additional average number of signal events $n_s = \sigma_s \cdot L$ for the integrated luminosity L. The total average number of the events is $< n >= n_s + n_b = (\sigma_s + \sigma_b) \cdot L$. So, as a result of new physics existence, we expect an excess of the average number of events. The probability of the realization of n events in experiment is described by Poisson distribution [1, 2]

$$f(n;\lambda) = \frac{\lambda^n}{n!}e^{-\lambda}. \tag{1}$$

In the report the approach to determination of the "significance" of predicted signal on new physics in concern to the predicted background is considered. This approach is based on the analysis of uncertainty [3, 4], which will take place under the future hypotheses testing about the existence of a new phenomenon in Nature. We consider a simple statistical hypothesis H_0: *new physics is present in Nature* (i.e. $\lambda = n_s + n_b$) against a simple alternative hypothesis H_1: *new physics is absent* ($\lambda = n_b$). The value of uncertainty is defined by the choosing of the critical value n_0, i.e. by Type I error α and Type II error β. Detailed description of the given investigation can be found in ref. [5].

2 "SIGNIFICANCE" IN PLANNED EXPERIMENT.

In the most of proposals of the experiments the following "significances" are used for testing the possibility to discover new physics:

(a) "significance" $S_1 = \dfrac{n_s}{\sqrt{n_b}}$ [6, 7],

(b) "significance" $S_2 = \dfrac{n_s}{\sqrt{n_s + n_b}}$ [8],

(c) "significance" $2 \cdot S_{12} = 2(\sqrt{n_s + n_b} - \sqrt{n_b})$ [9, 3].

As shown [4, 10] the "significance" $2 \cdot S_{12}$ more proper in planned experiments. Note, all these "significances" assume a 50% acceptance for positive decision about new physics observation.

If we define the "signal significance" according to ref. [11] as "effective significance" s

$$\frac{1}{\sqrt{2\pi}} \int_s^{\infty} exp(-x^2/2)dx = \sum_{k=n_0}^{\infty} f(k; n_b), \tag{2}$$

where n_0 is the critical value for hypotheses testing, the system

$$\beta = \sum_{n=n_0+1}^{\infty} f(n; n_b) \leq \Delta \tag{3}$$

$$1 - \alpha = \sum_{n=n_0+1}^{\infty} f(n; n_s + n_b) \tag{4}$$

allows us to construct dependences n_s versus n_b on given value of Type II error $\beta \leq \Delta$ and given acceptance $1-\alpha$. If $\Delta = 2.85 \cdot 10^{-7}$ ($s \geq 5$, i.e. the value n_0 has 5σ deviation from average background n_b), the corresponding acceptance may be named *the probability of discovery*; if $\Delta = 0.0014$ ($s \geq 3$), the acceptance is *the probability of strong evidence*, and, if $\Delta = 0.028$ ($s \geq 2$), the acceptance is *the probability of weak evidence.*

For the estimation of "effective significance" s with given acceptance $1 - \alpha$ approximate formulae can be used:

$$s = S_1 - k(\alpha)\sqrt{1 + \frac{n_s}{n_b}} \tag{5}$$

or

$$s = 2 \cdot S_{12} - k(\alpha). \tag{6}$$

where $k(\alpha)$: $k(0.5) = 0$; $k(0.25) = 0.66$; $k(0.1) = 1.28$; $k(0.05) = 1.64$ (for instance, Tab.28.1 [1]).

3 EXCLUSION LIMITS [3, 4]

It is important to know the range in which a planned experiment can exclude presence of signal at given confidence level $(1 - \epsilon)$. It means that we will have uncertainty in future hypotheses testing about non-observation of signal which equals to or less than ϵ. In refs.[12, 13] different methods to derive exclusion limits in prospective studies have been suggested.

We propose to use the relative uncertainty

$$\tilde{\kappa} = \frac{\alpha + \beta}{2 - (\alpha + \beta)} \tag{7}$$

which will take place under hypotheses testing H_0 versus H_1. This relative uncertainty $\tilde{\kappa}$ in case of applying the equal-probability test [4] is a minimal relative value of the number of wrong decisions in the future hypotheses testing for Poisson distributions. It is the uncertainty in the observability of the new phenomenon. Note that $1 - \tilde{\kappa}$ (the relative number of correct decisions) may be considered as a distance between two distributions (the measure of distinguishability of two Poisson processes) in frequentist sense.

4 AN ACCOUNT OF SYSTEMATIC UNCERTAINTY RELATED TO NONEXACT KNOWLEDGE OF BACKGROUND AND SIGNAL CROSS SECTIONS [3]

In ref. [14], for instance, the systematic uncertainty is the uncertainty in the sensitivity factor. This uncertainty has statistical properties which can be measured or estimated.

We consider here forthcoming experiments to search for new physics. In this case we must take into account the systematic uncertainty which have theoretical origin without any statistical properties. For example, two loop corrections for most reactions at present are not known. It means that we can only estimate the scale of influence of background uncertainty on the observability of signal, i.e. we can point the admissible level of uncertaity in theoretical calculations for given experiment proposal.

Suppose uncertainty in the calculation of exact background cross section is determined by parameter δ, i.e. the exact cross section lies in the interval $(\sigma_b, \sigma_b(1+\delta))$ and the exact value of average number of background events lies in the interval $(n_b, n_b(1+\delta))$. Let us suppose $n_b \gg n_s$. In this instance the discovery potential is the most sensitive to the systematic uncertainties. As we know nothing about possible values of average number of background events, we consider the worst case. Taking into account formulae (3) and (4) we have the formulae [1]

$$\beta = \sum_{n=n_0+1}^{\infty} f(n; n_b(1+\delta)) \leq \Delta \tag{8}$$

$$1 - \alpha = \sum_{n=n_0+1}^{\infty} f(n; n_b + n_s) \tag{9}$$

5 CONCLUSIONS

In this paper we have described a method to estimate the discovery potential and exclusion limits on new physics in planned experiments where only the average number of background n_b and signal events n_s is known. The “effective significance” s of signal for given probability of observation is discussed. We also estimate the influence of systematic uncertainty related to nonexact knowledge of signal and background cross sections on the probability to discover new physics in planned experiments. An account of such kind of systematics is very essential in the search for supersymmetry and leads to an essential decrease in the probability to discover new physics in future experiments. The texts of programs can be found in **http://home.cern.ch/bityukov**.

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